

LA-UR-20-30087

Approved for public release; distribution is unlimited.

Title: West Antarctic Ice Sheet history and dynamics

Author(s): Hillebrand, Trevor Ray
Siddoway, Christine

Intended for: Lecture slides for course taught at Colorado College

Issued: 2020-12-08

Disclaimer:

Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by Triad National Security, LLC for the National Nuclear Security Administration of U.S. Department of Energy under contract 89233218CNA000001. By approving this article, the publisher recognizes that the U.S. Government retains nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

West Antarctic Ice Sheet history and dynamics

Lecture slides for GY400 – Senior Seminar in Geology
Colorado College, Fall 2020, Block 3

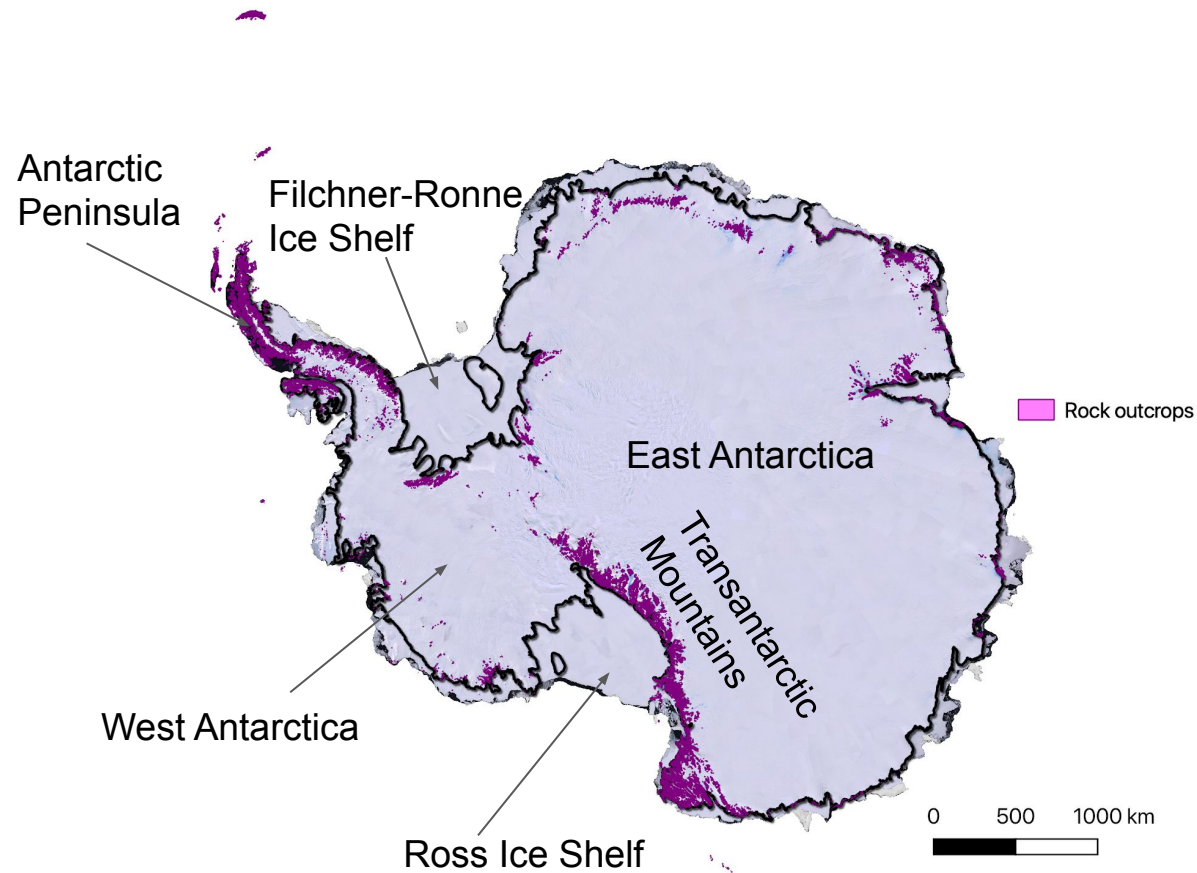
Co-instructors:

Dr. Trevor R Hillebrand — Los Alamos National Laboratory

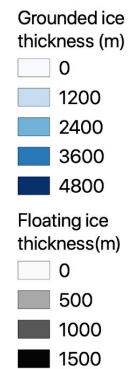
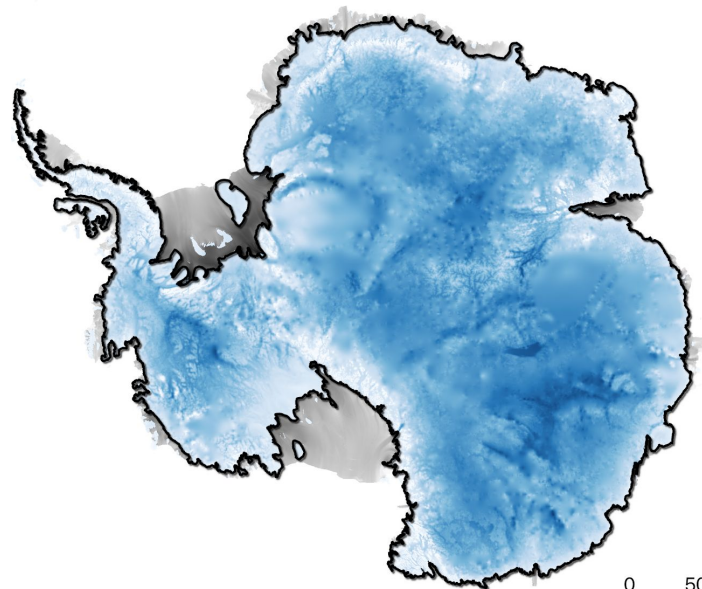
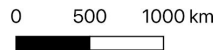
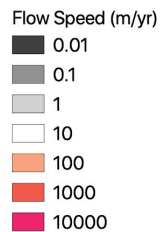
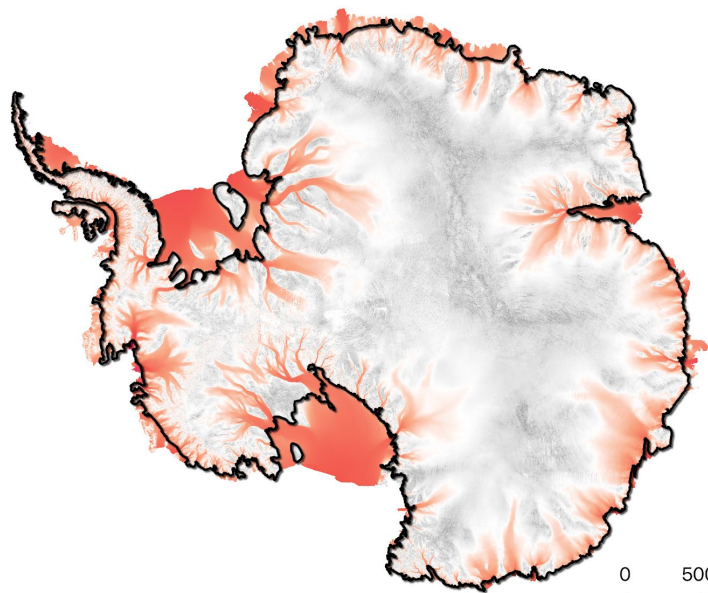
Dr. Christine Siddoway — Colorado College

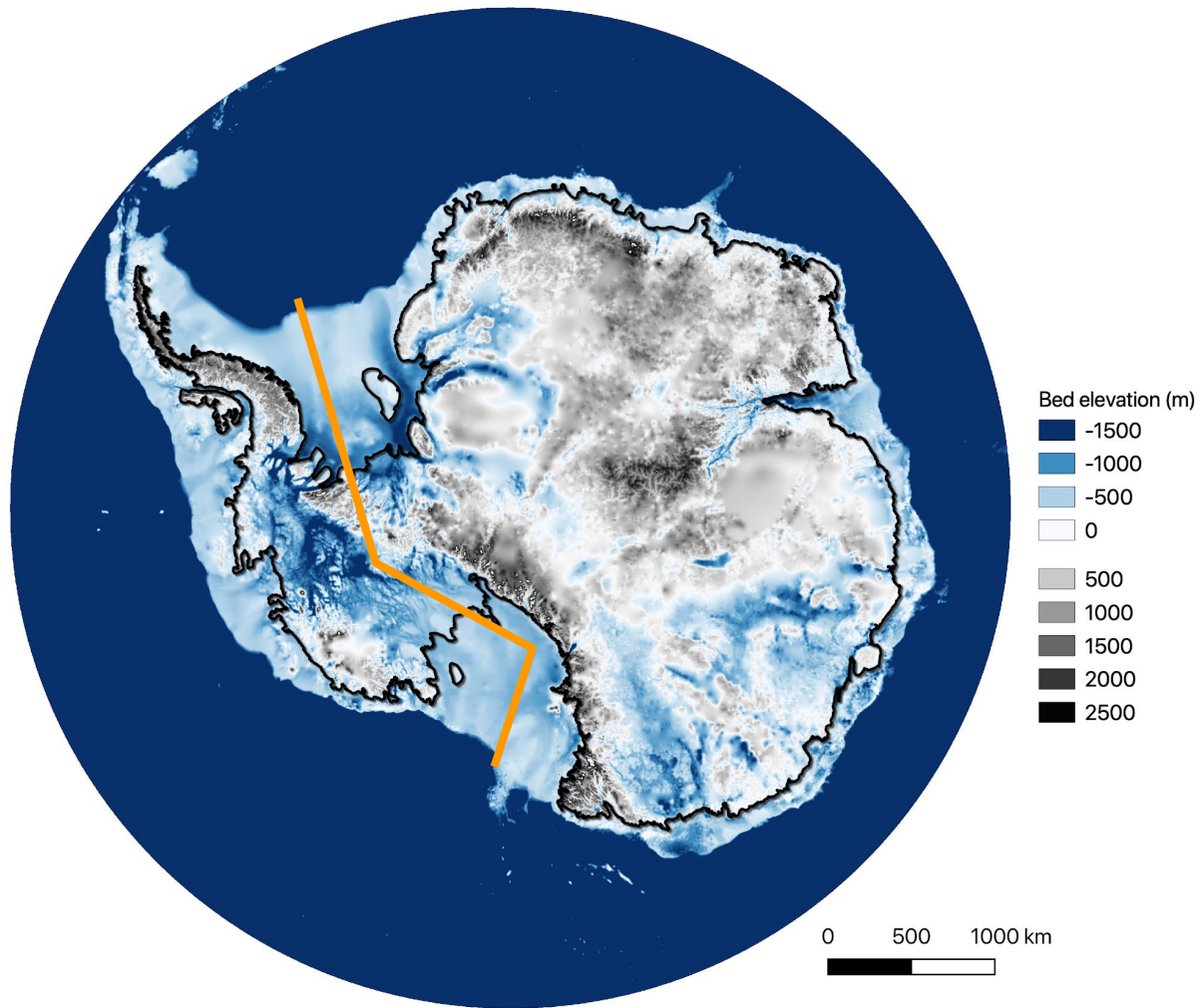
The West Antarctic Ice Sheet: a primer

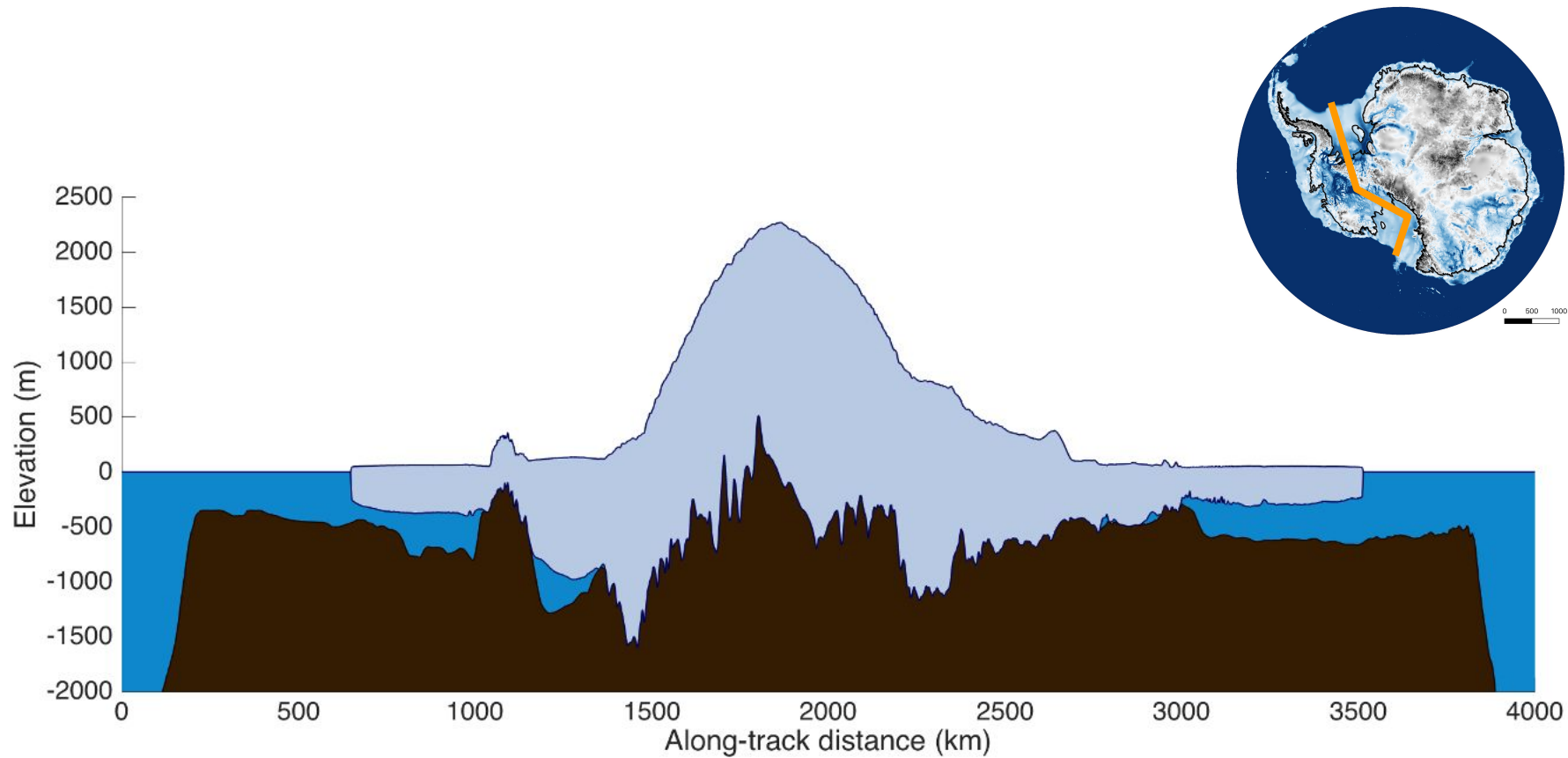


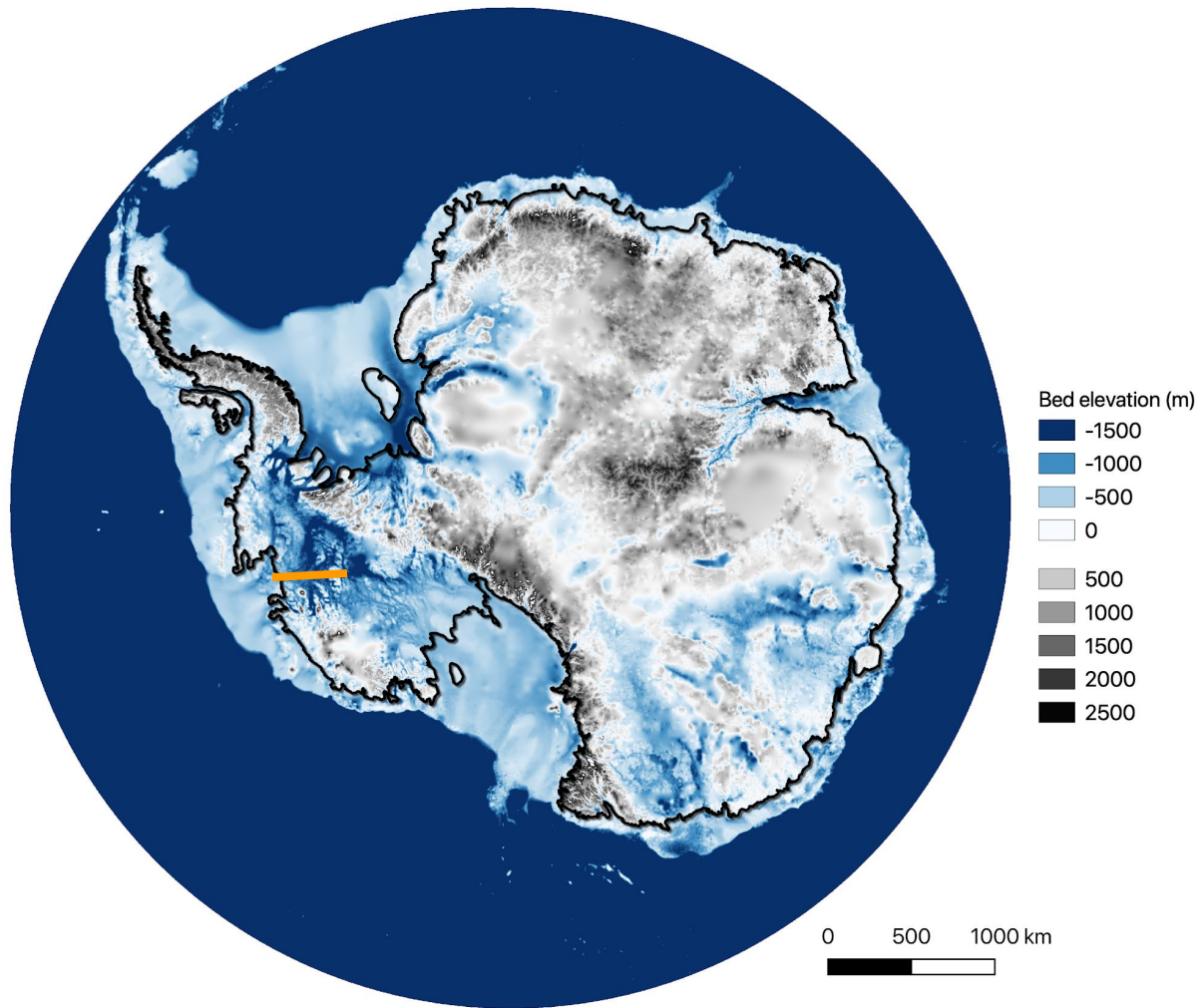


<1 % of Antarctica is ice-free

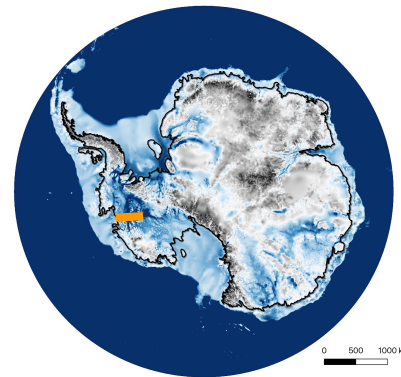
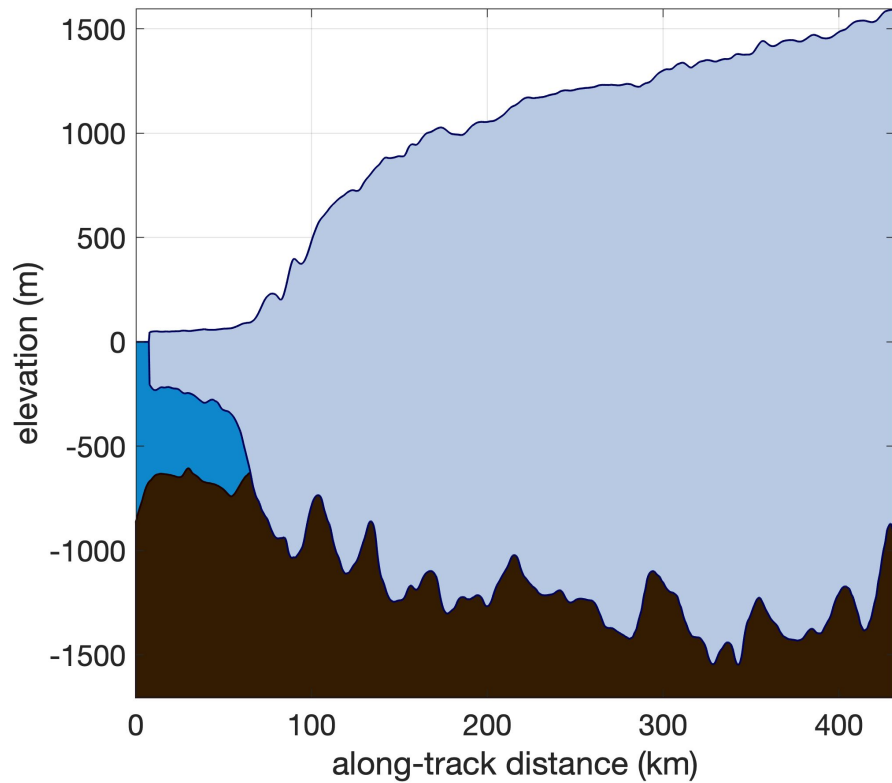




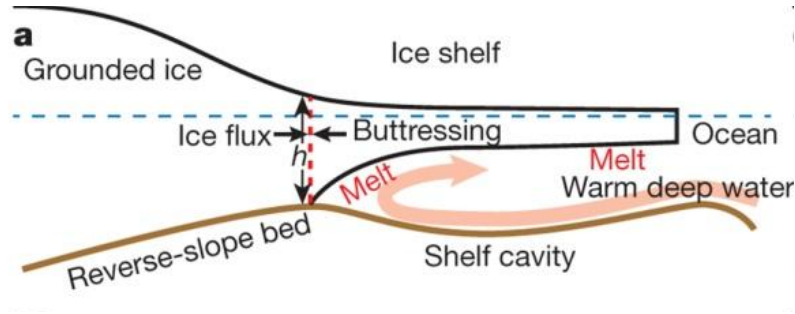




The bed of West Antarctica deepens inland for hundreds of kilometers. That is not great for ice-sheet stability.

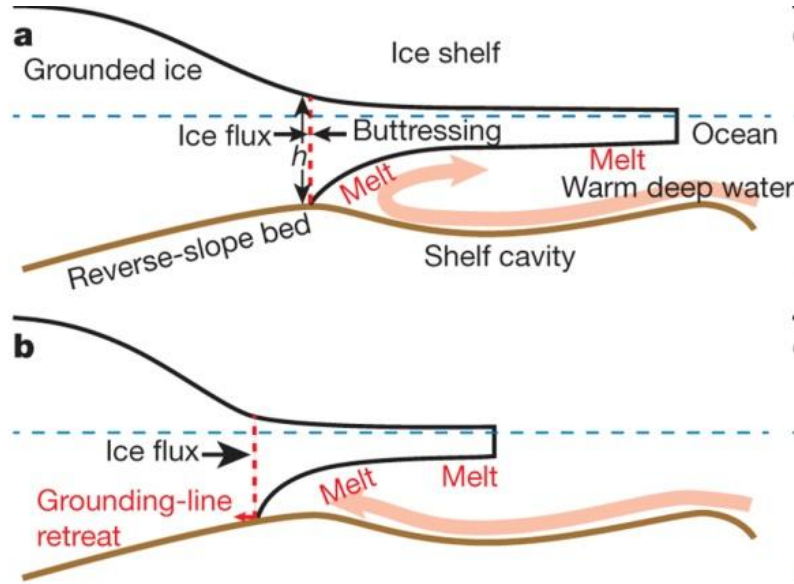


Instability due to ice flow



$$\text{Ice flux} \propto (\text{ice thickness at grounding line})^5$$

Instability due to ice flow



Ice flux \propto (ice thickness at grounding line)⁵

Increased melting

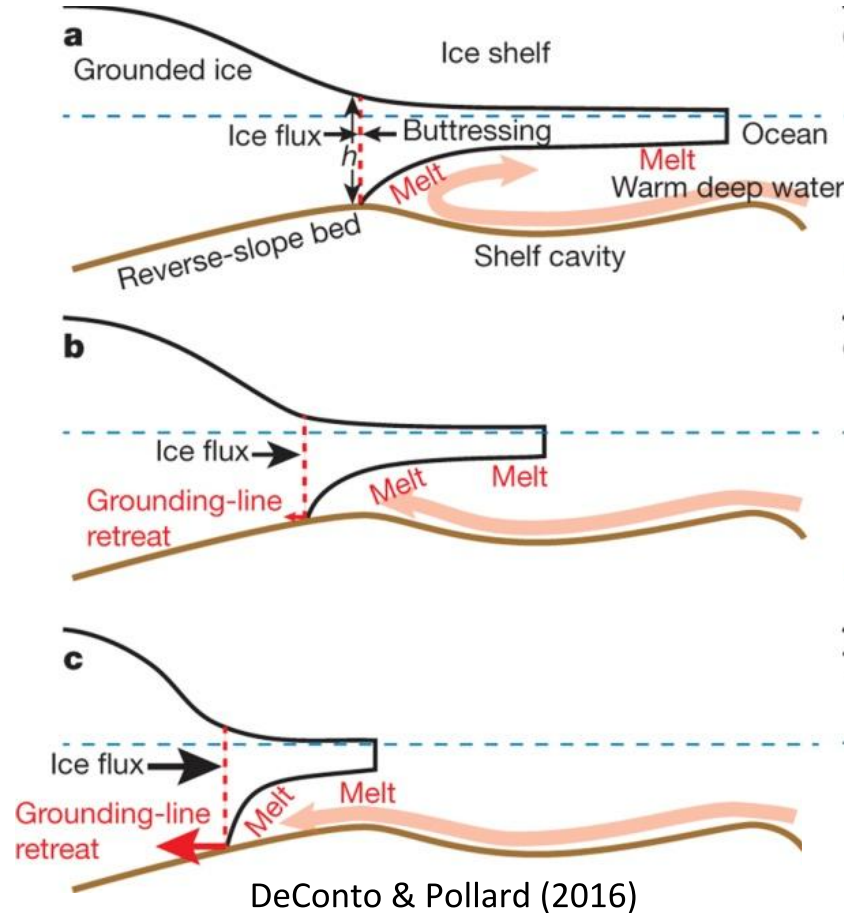
→ Grounding-line retreats into thicker ice

→ Increases ice flux

→ Accelerates and thins ice upstream

→ Further grounding-line retreat

Instability due to ice flow



$$\text{Ice flux} \propto (\text{ice thickness at grounding line})^5$$

Increased melting

→ Grounding-line retreats into thicker ice

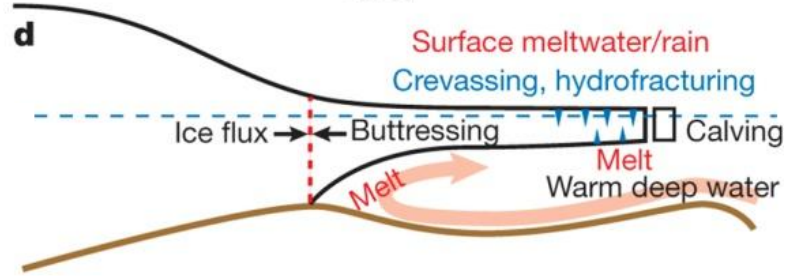
→ Increases ice flux

→ Accelerates and thins ice upstream

→ Further grounding-line retreat

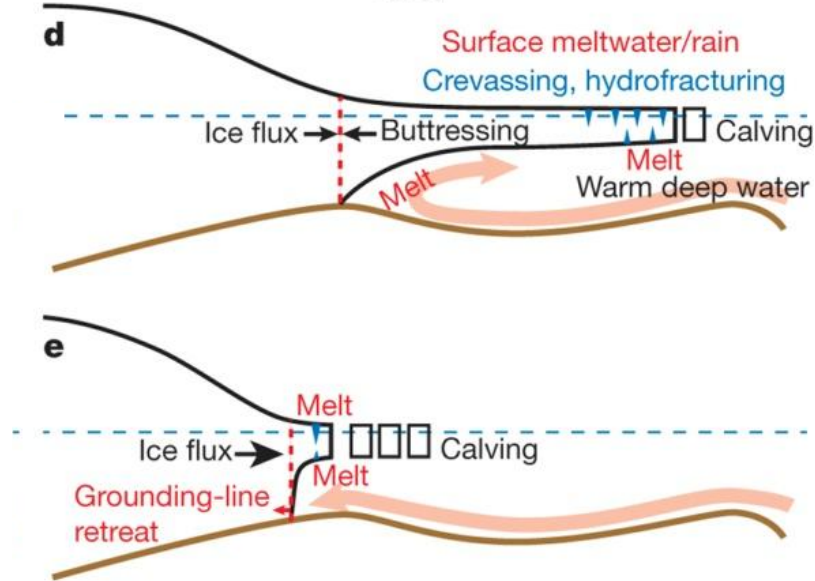


Instability due to ice fracture



Surface melting causes
hydrofracture of ice shelves

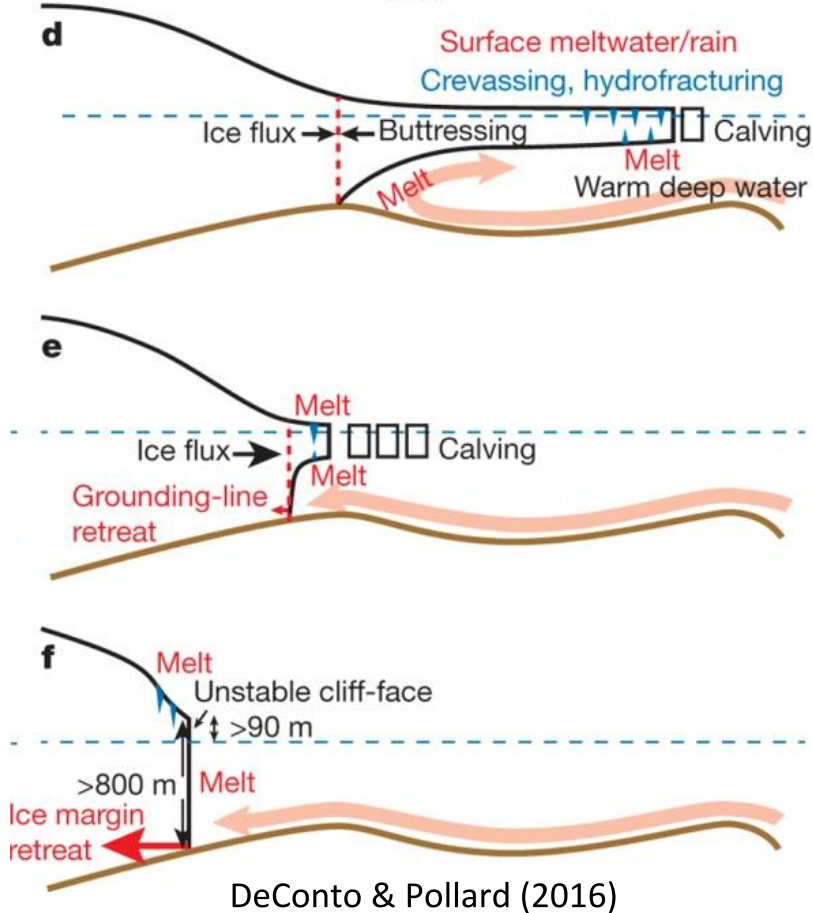
Instability due to ice fracture



Increased surface melting
→ hydrofracture of ice shelves

→ Rapid loss of ice shelves
causes grounded ice to
accelerate

Instability due to ice fracture



Increased surface melting
→ hydrofracture of ice shelves

→ Rapid loss of ice shelves
causes grounded ice to
accelerate

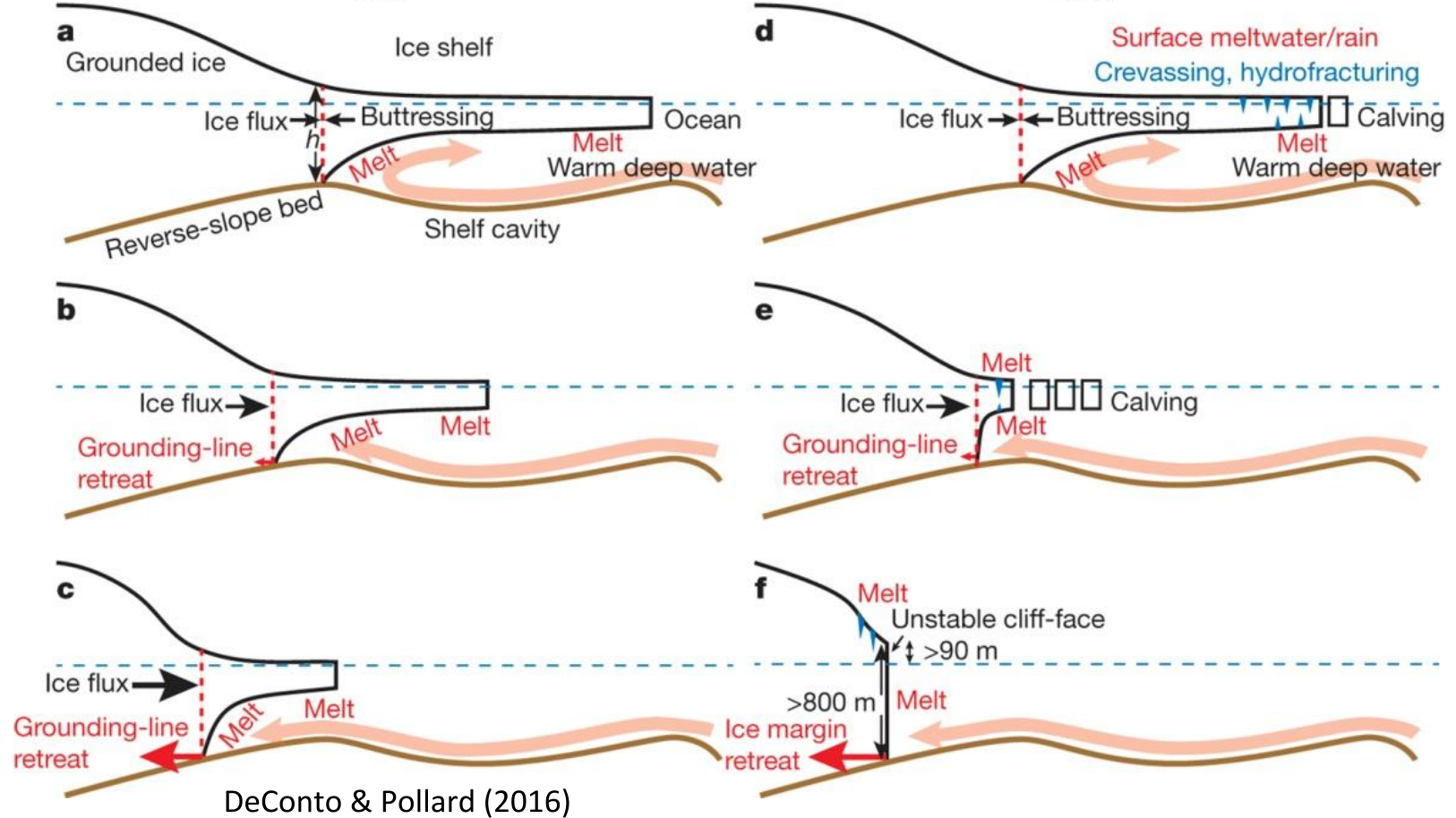
→ Cliff at ice edges reaches
unstable height and collapses

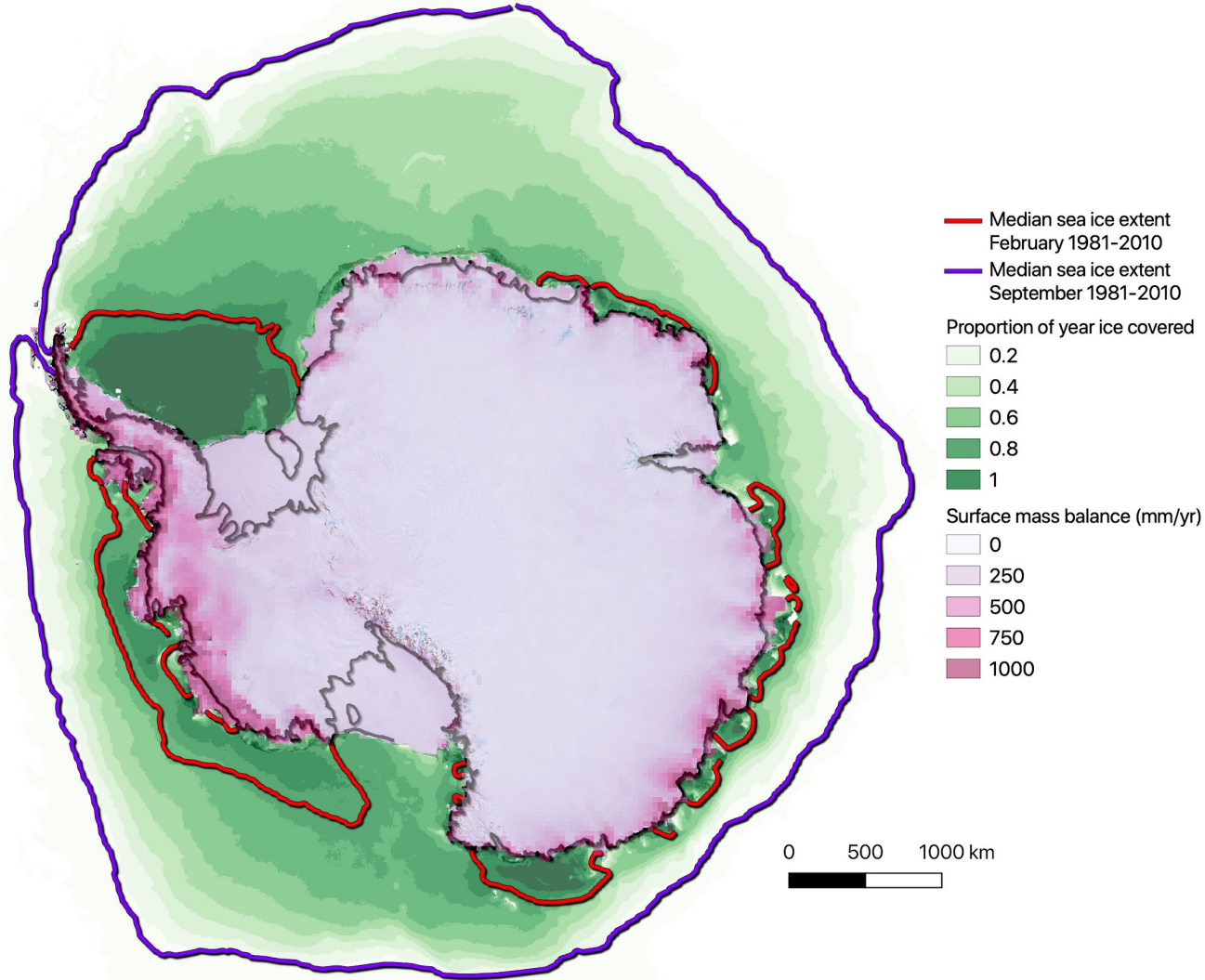
→ Margin retreats into thicker
ice, forming higher cliff



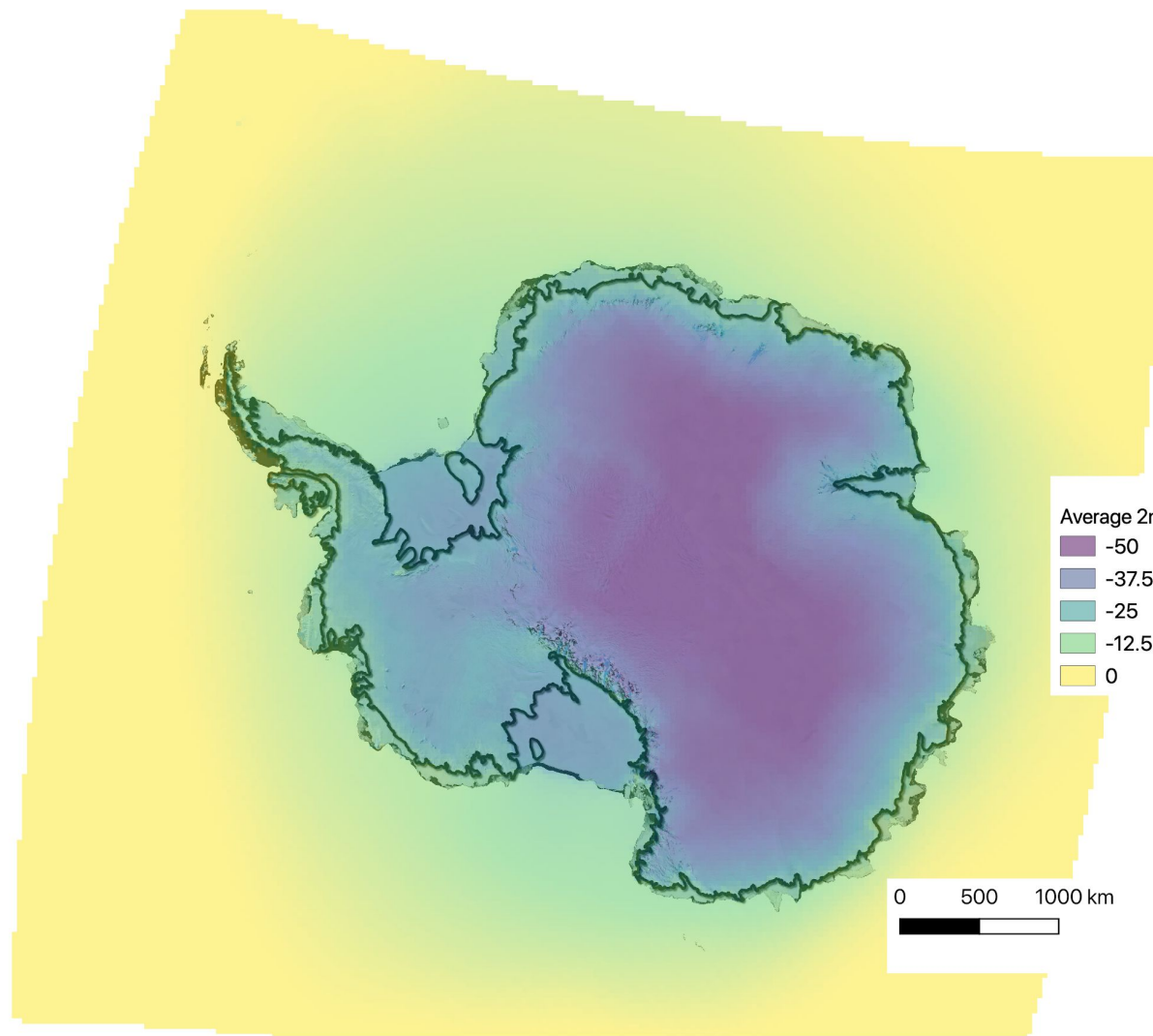
Marine Ice Sheet Instability: ~ 1 km/yr

Marine Ice Cliff Instability: 10 km/yr (?)





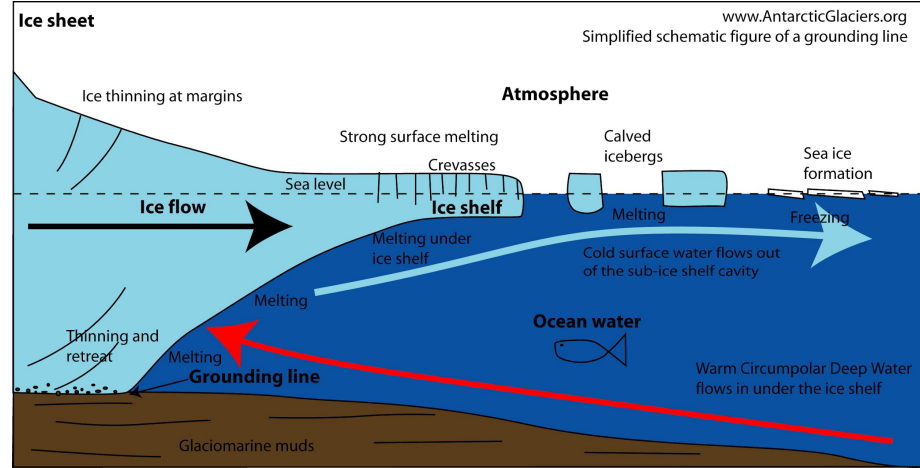
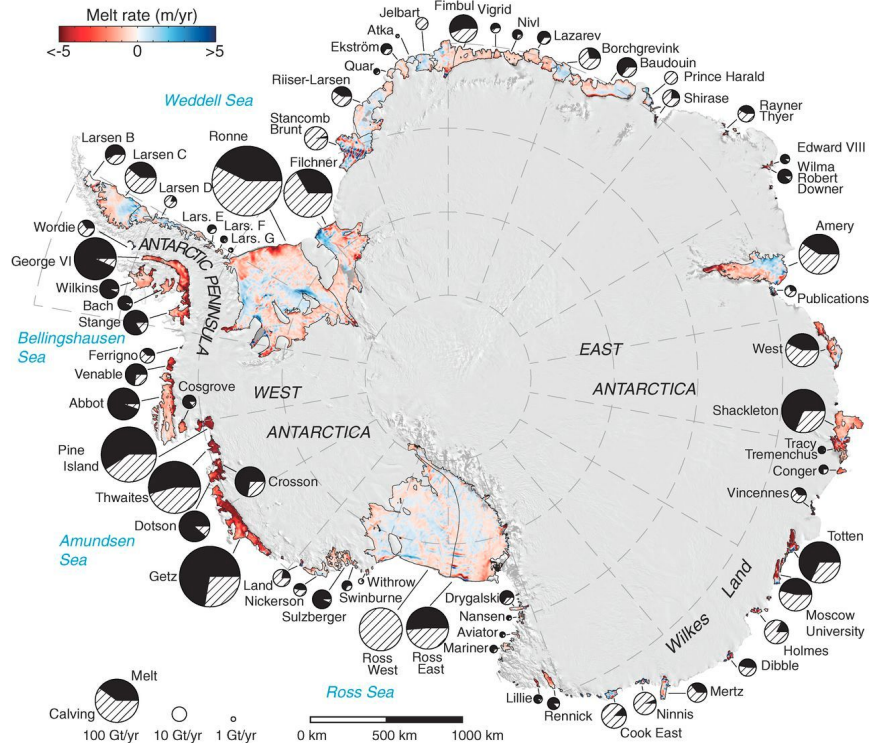
Cold air holds little
moisture, so snowfall
rates are low.



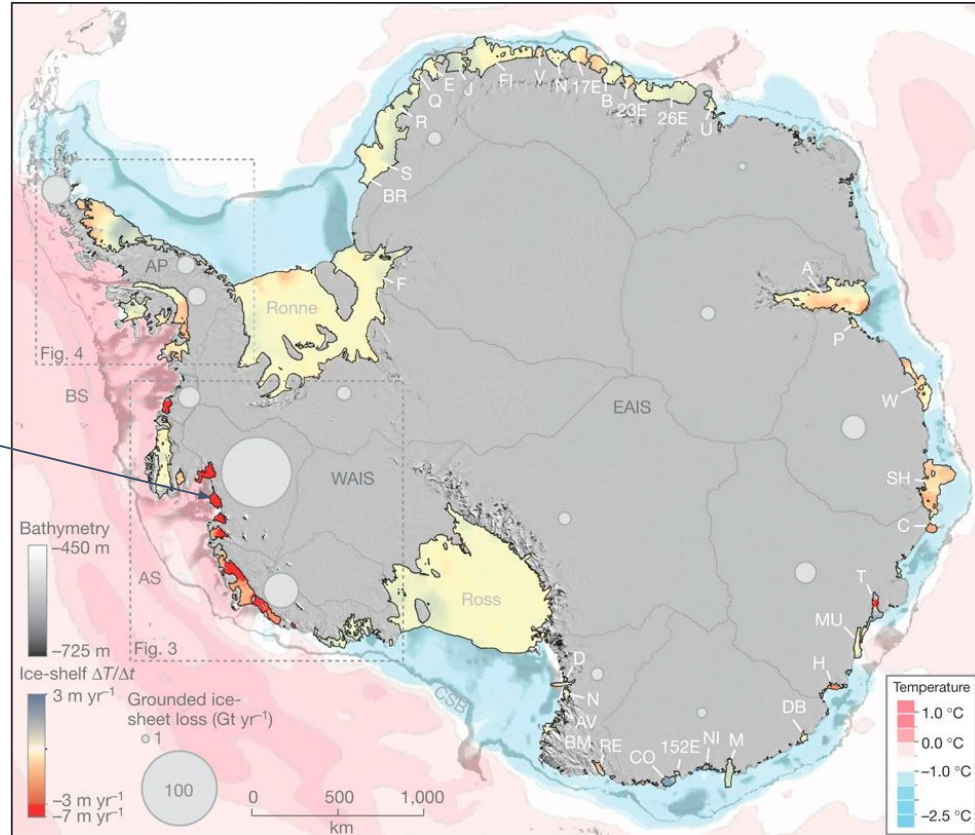
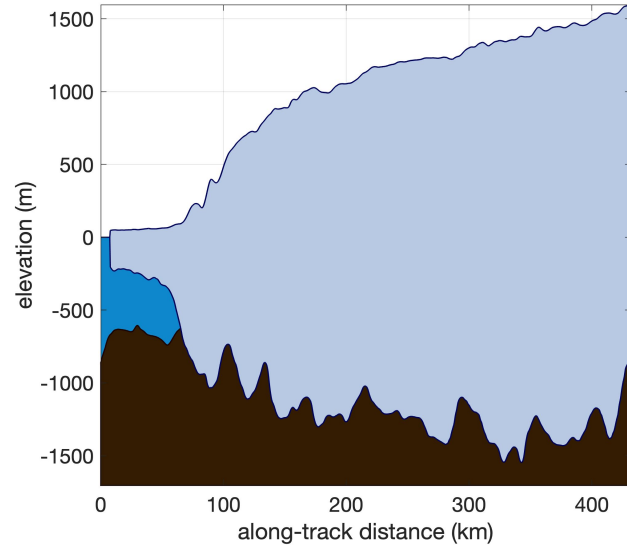
Temperatures
almost never rise
above freezing.

Mass loss by
surface melt is
currently
negligible.

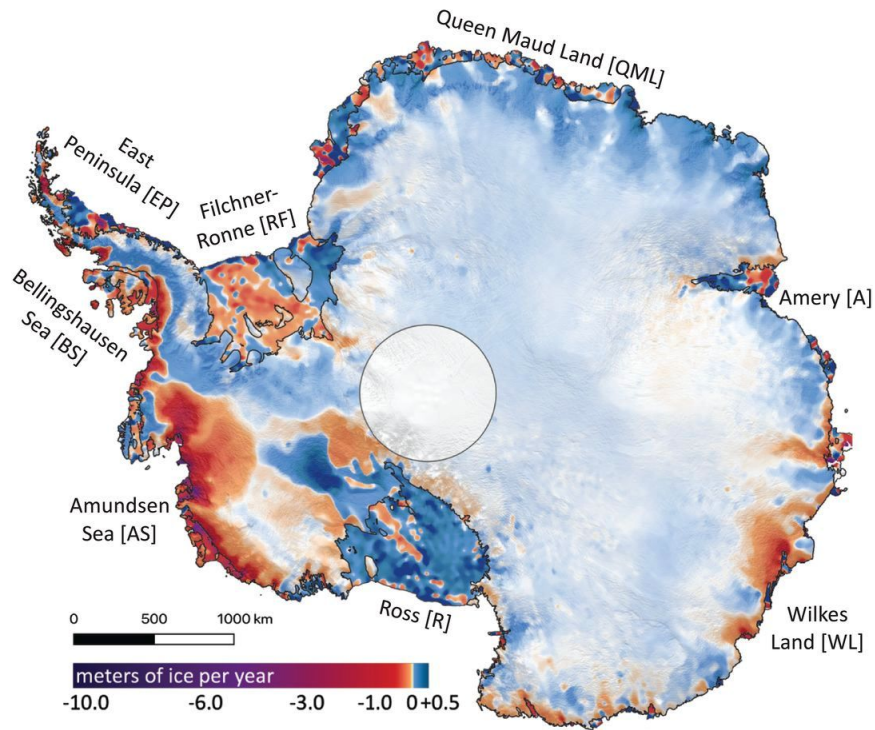
Mass loss from Antarctica is almost entirely from melting and calving of ice shelves



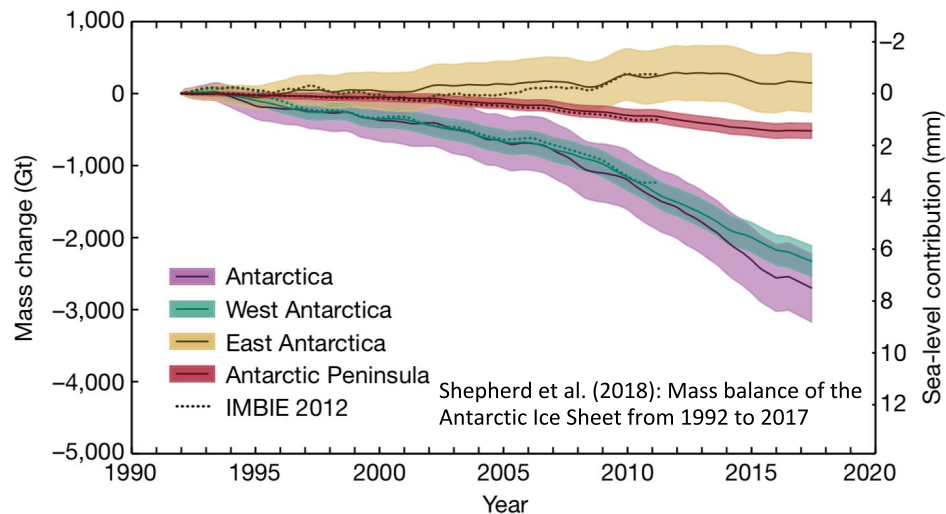
Warm ocean water is able to access the grounding-line of Thwaites Glacier



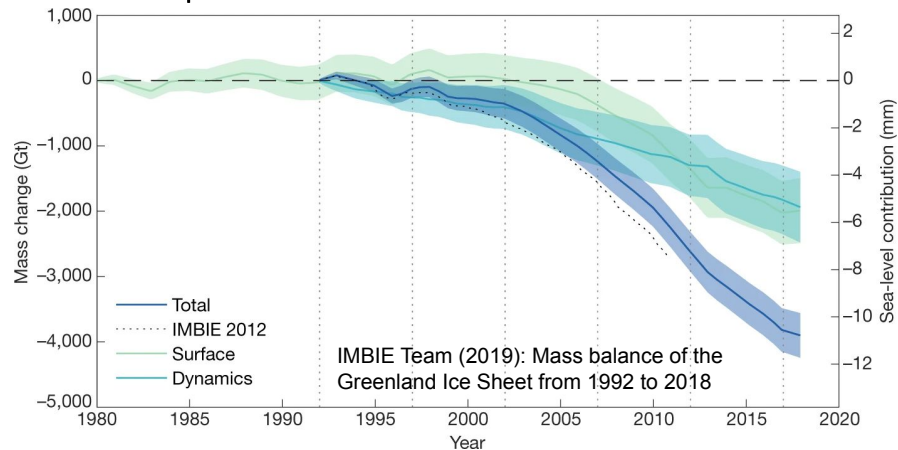
Present day changes



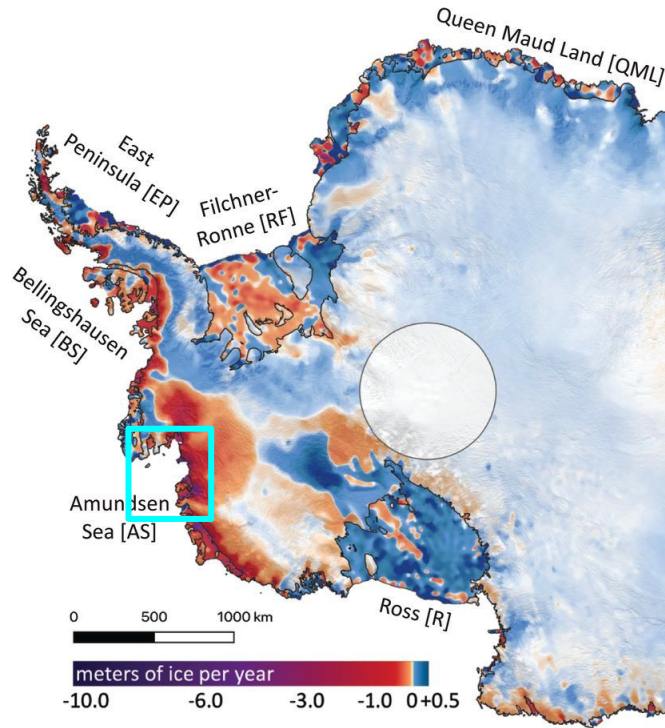
Smith et al. (2020): Pervasive ice sheet mass loss reflects competing ocean and atmosphere processes



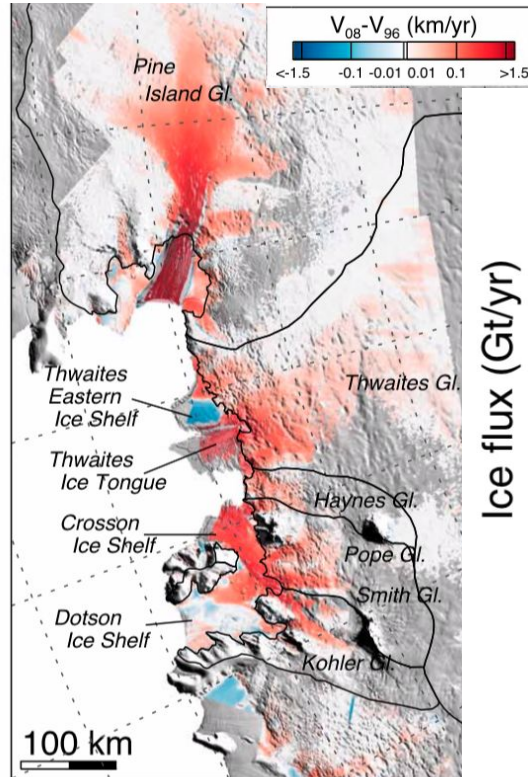
Compare to Greenland:



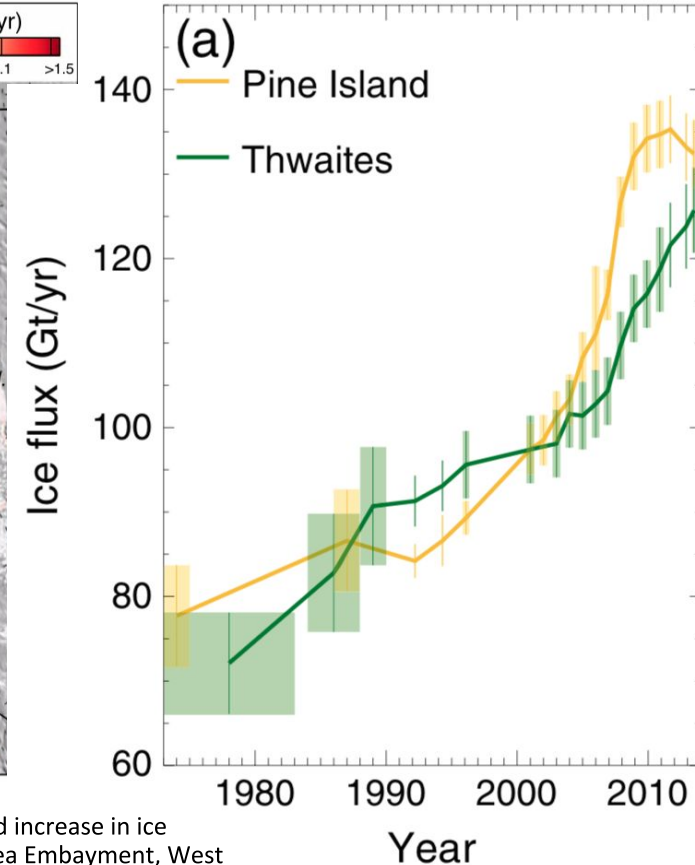
Present day changes



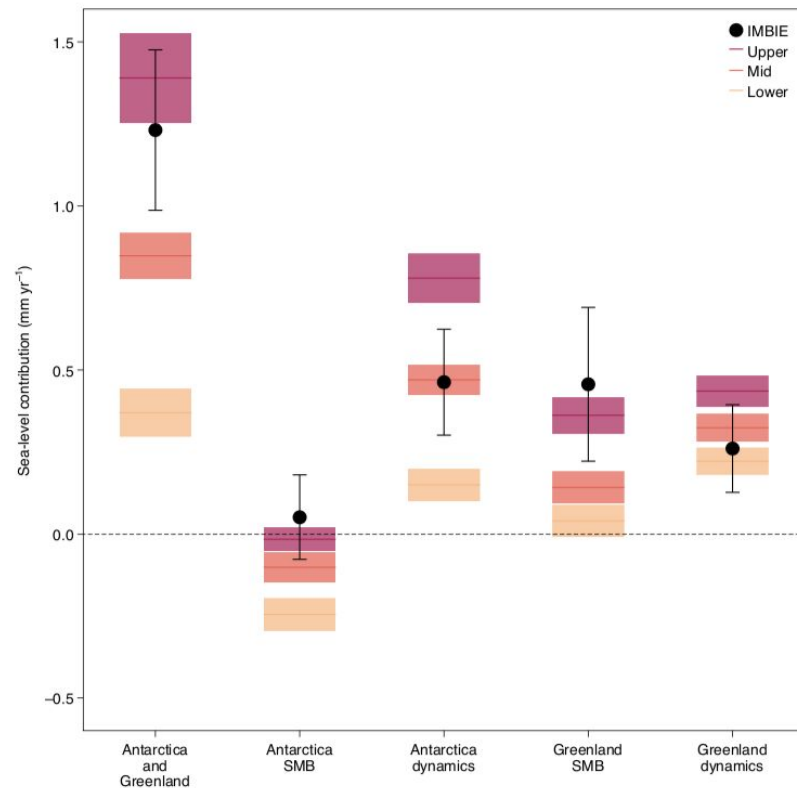
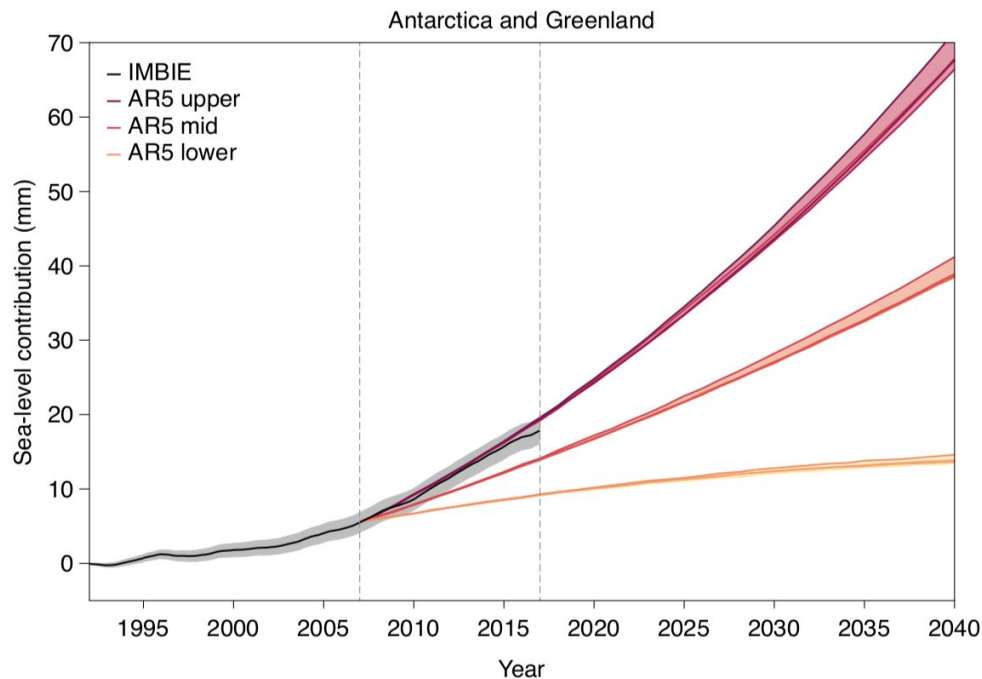
Smith et al. (2020): Pervasive ice sheet mass loss reflects competing ocean and atmosphere processes



Mouginot et al. (2014): Sustained increase in ice discharge from the Amundsen Sea Embayment, West Antarctica, from 1973 to 2013

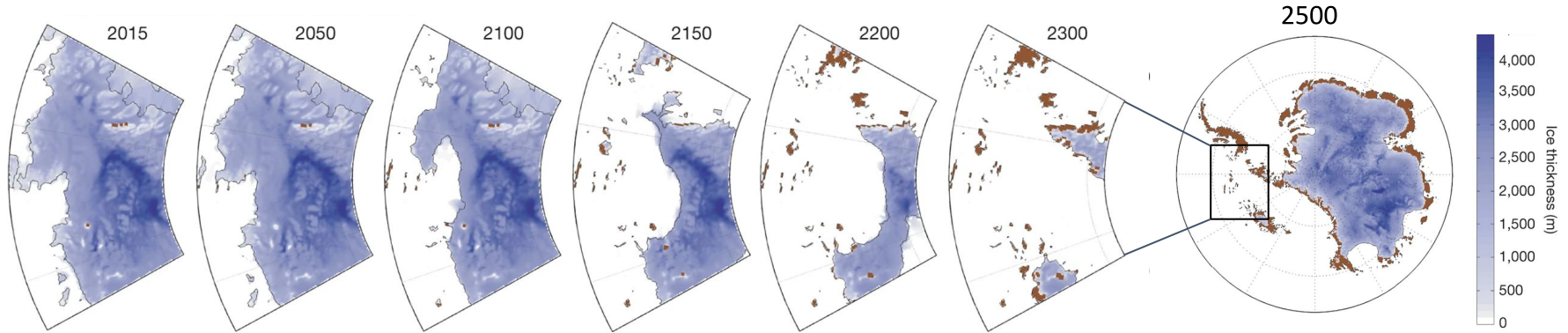


Present day changes

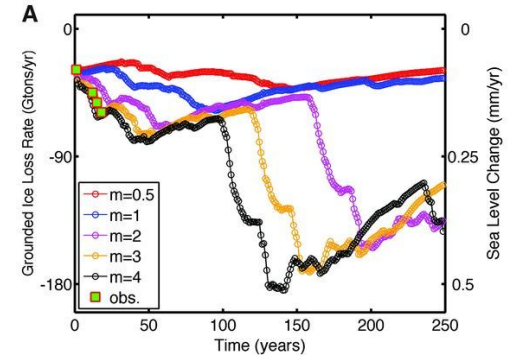
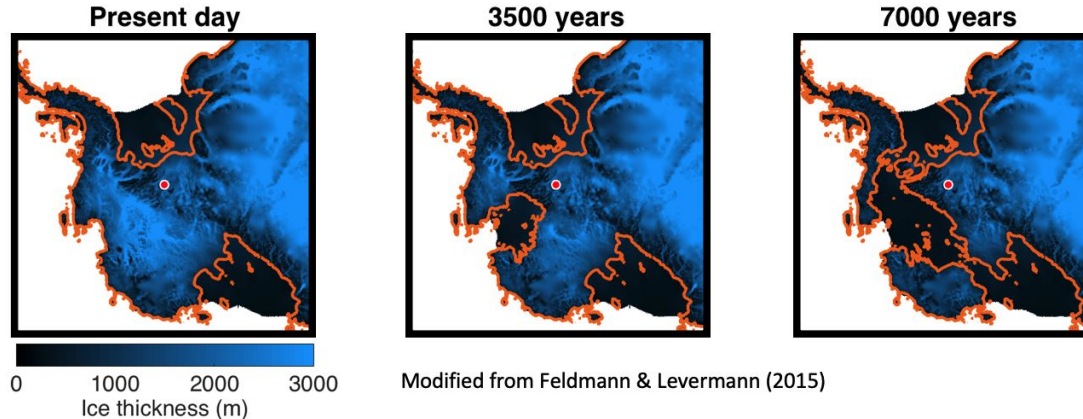


Slater, Hogg, and Mottram (2020): Ice-sheet losses track high-end sea-level rise projections

Multiple models predict future deglaciation of West Antarctica, with Thwaites Glacier. The rate and timing are unknown.



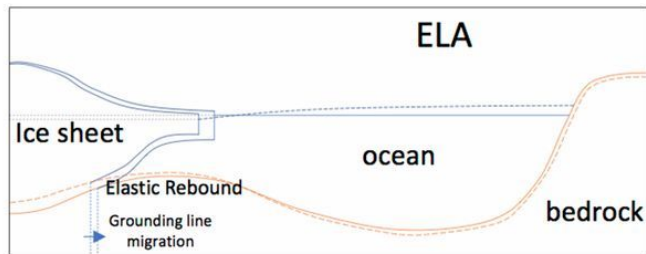
DeConto & Pollard (2016)



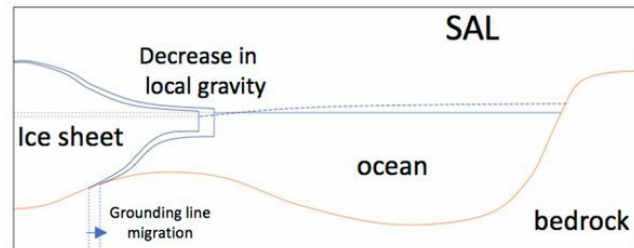
Joughin et al. (2014)

Possible mitigating factors: solid earth and sea-level feedbacks

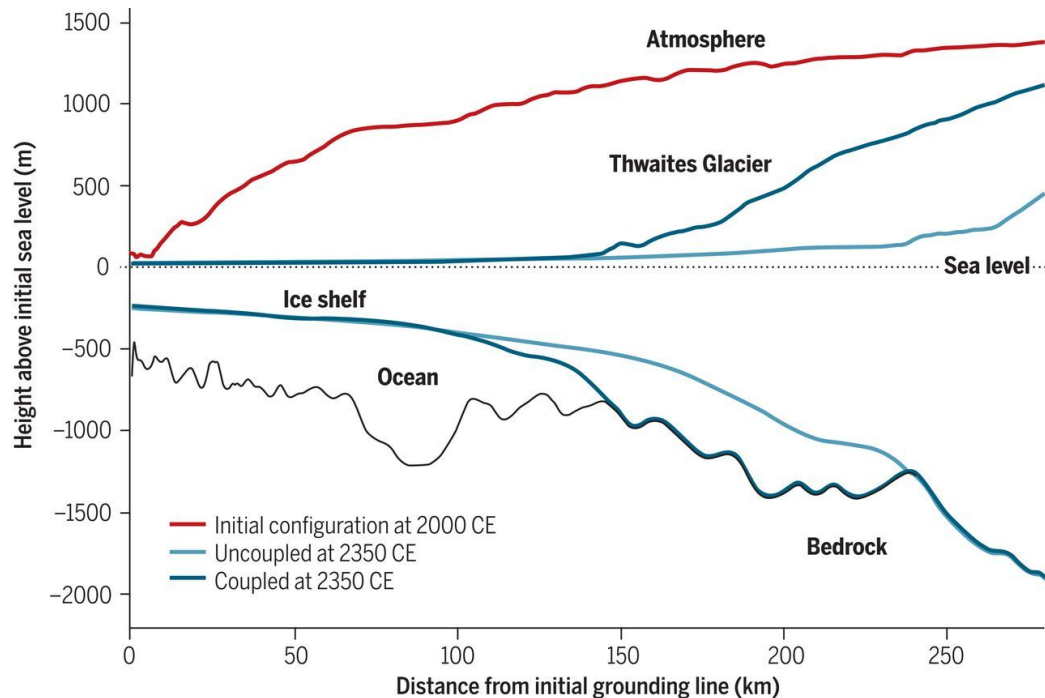
As the ice sheet thins, the bed rebounds elastically.



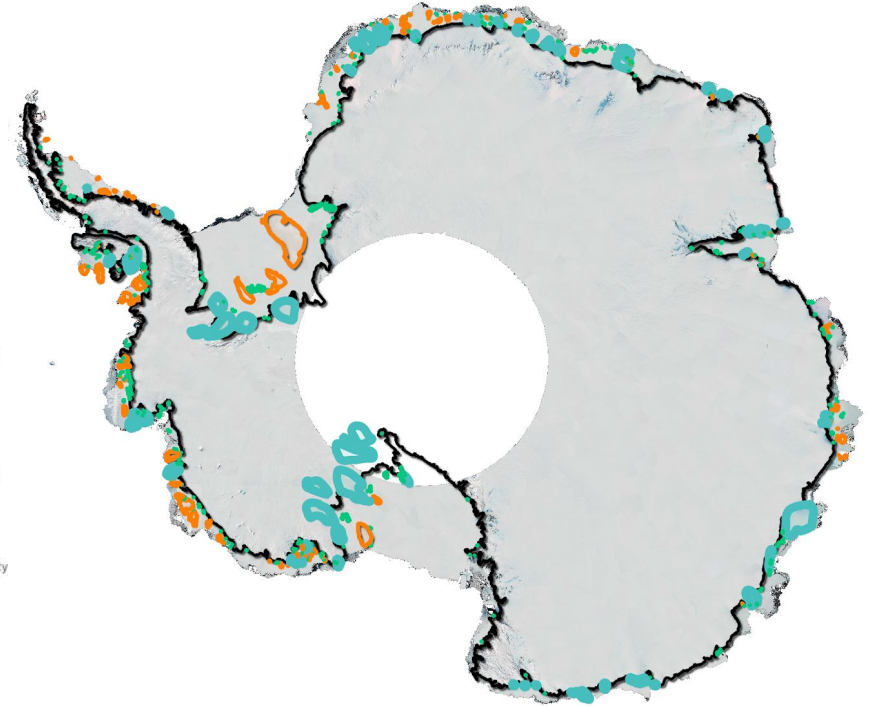
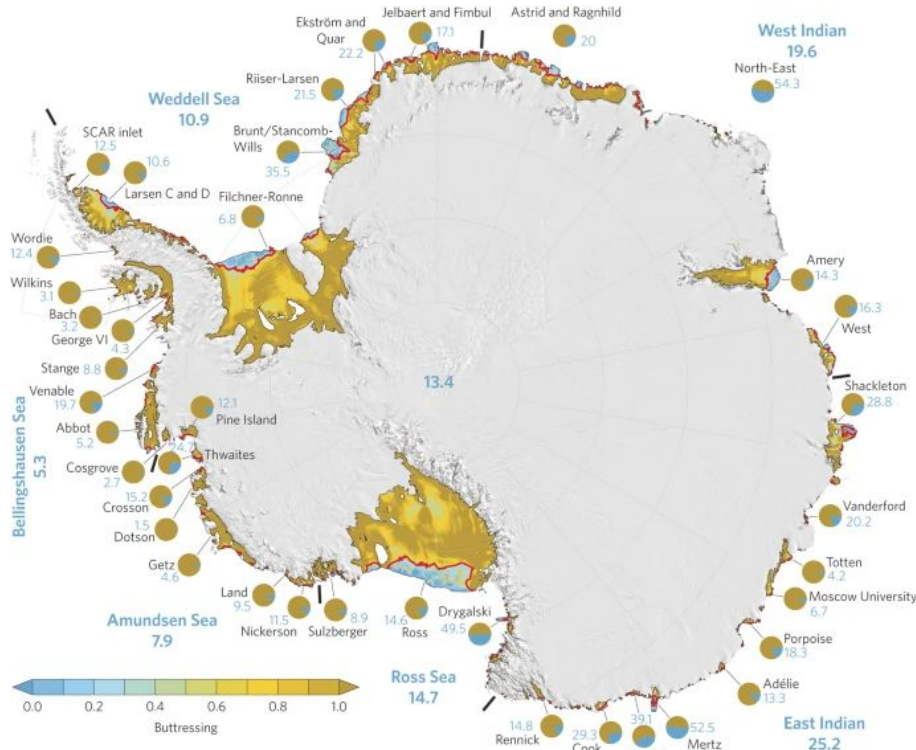
And the gravitational pull on the ocean decreases.



The effect is small for the 21st century, but multi-century projections could overestimate mass loss by 20–40% if they do not account for these feedbacks.



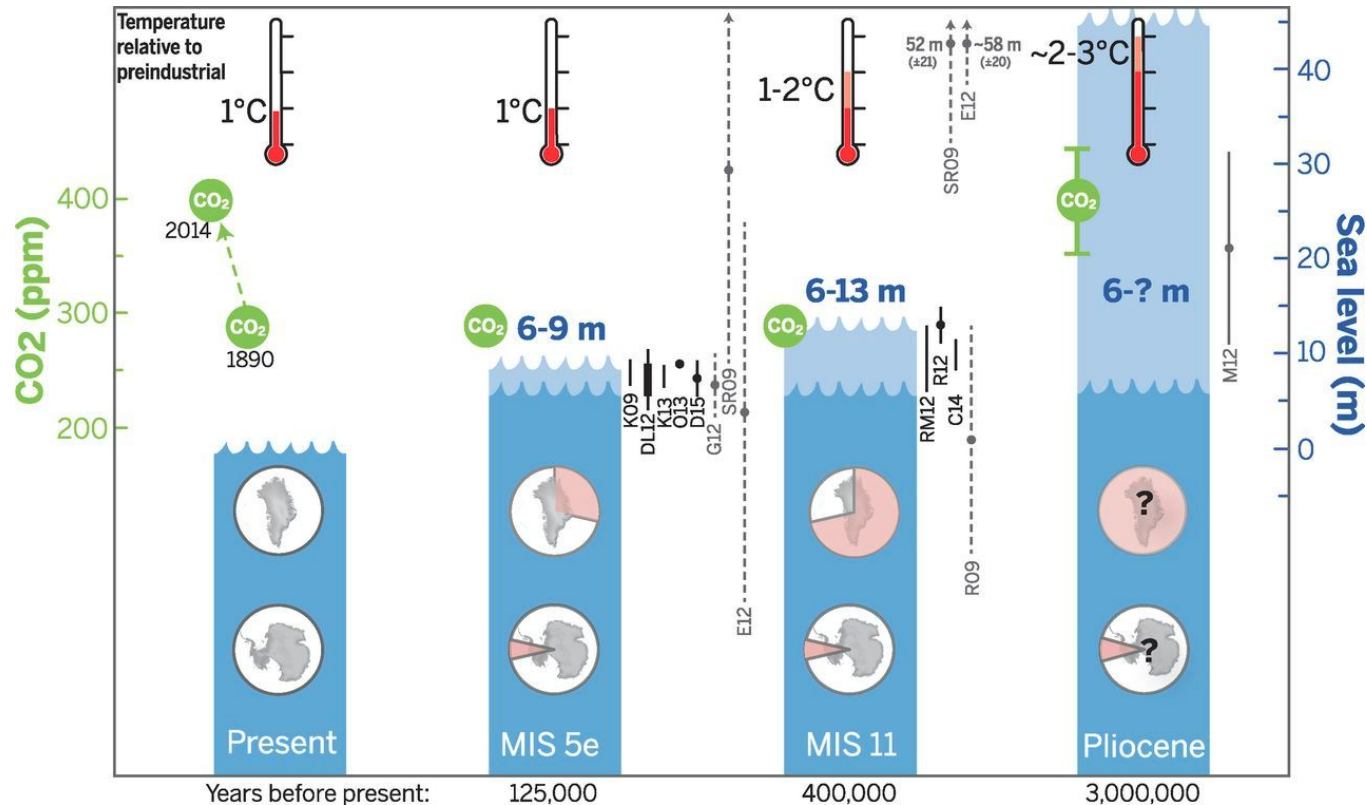
Possible mitigating factors: buttressing by ice shelves and ice rises



“Buttressing” is a pressure exerted against ice flow, like a flying buttress supporting a cathedral.

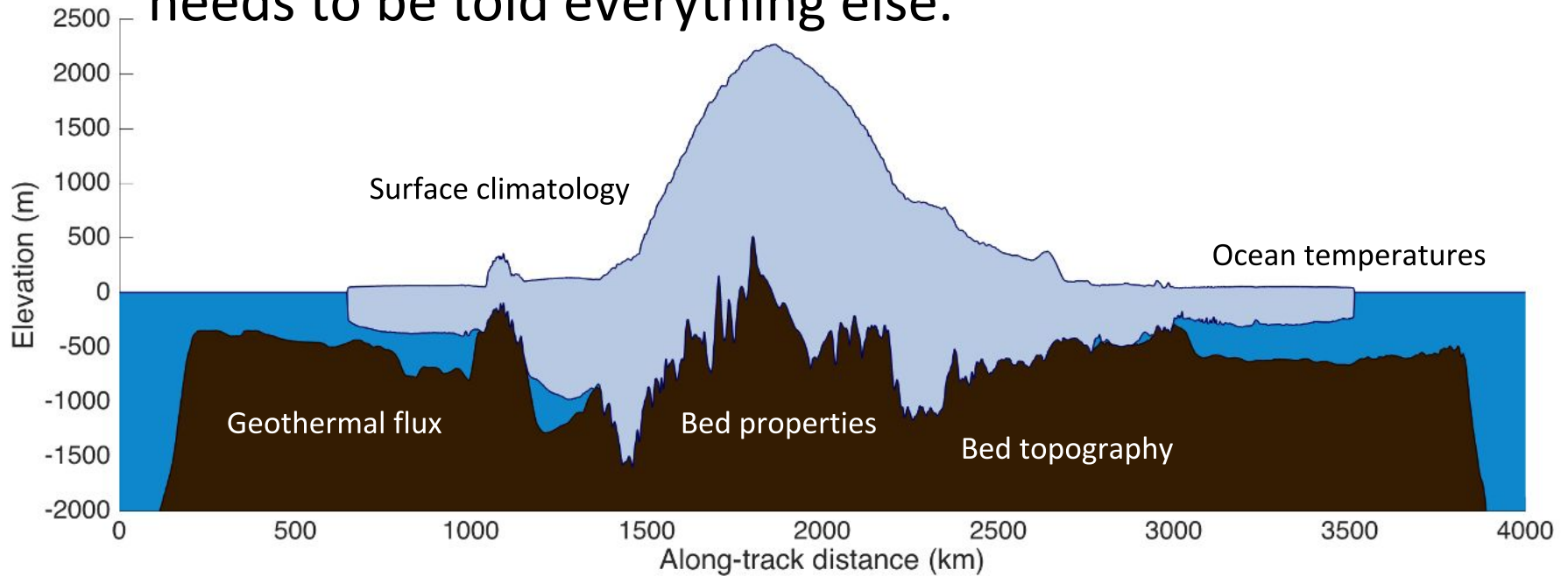
When has the West Antarctic Ice Sheet collapsed in the past?

What are the climate thresholds for collapse?



What do we mean by boundary conditions?

An ice sheet model handles the physics of ice flow, but it needs to be told everything else.



Introduction to ice dynamics



Mass conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

$$\frac{\partial \rho}{\partial t} = 0; \frac{\partial \rho}{\partial x_i} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy} + \dot{\epsilon}_{zz} = 0$$

Ice is incompressible with close to homogeneous density

With the constraint of constant, uniform density, mass continuity equates to volume continuity.

We often express these velocity gradients as strain rates.

Mass conservation and evolution of ice thickness

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \longrightarrow \frac{\partial w}{\partial z} = - \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

Rearrange to solve in the vertical dimension.

$$\int_0^H \frac{\partial w}{\partial z} dz = - \int_0^H \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz = - \left(\frac{\partial}{\partial x} \int_0^H u dz + \frac{\partial}{\partial y} \int_0^H v dz \right)$$

Integrate over entire ice thickness

$$w|_{z=H} - w|_{z=0} = - \left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \right) H = -\nabla \cdot \mathbf{Q}$$

at $z=0$, $w=0$

Get vertical velocity (w) in terms of depth-averaged horizontal flux gradients.

$$w = \frac{\partial h}{\partial t} \longrightarrow \frac{\partial H}{\partial t} = \underbrace{-\nabla \cdot \mathbf{Q}}_{\text{Flux divergence: deformation due to velocity gradients}} + \underbrace{a_s}_{\text{surface mass balance: net rate of snow accumulation or melting}} - \underbrace{a_b}_{\text{basal mass balance: net rate of melting or freezing}}$$

Express thickness change as a function of flux divergence and source terms (snowfall, melting)

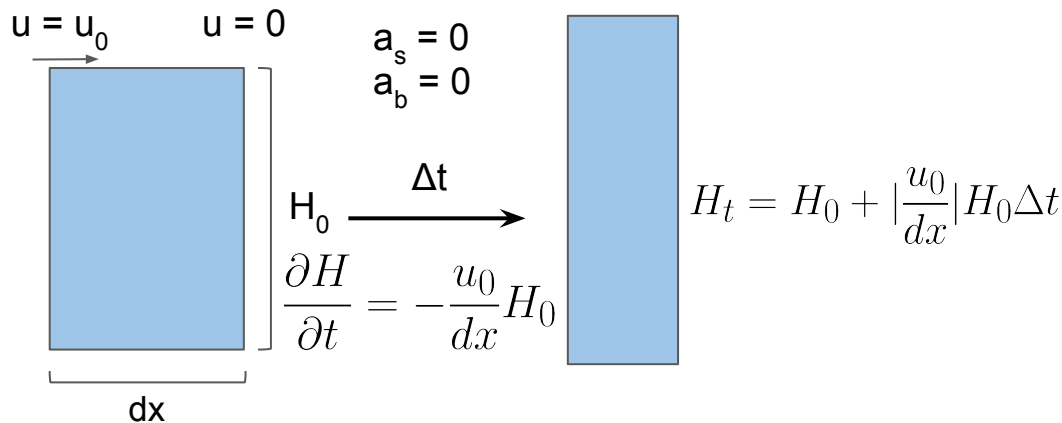
Mass conservation and evolution of ice thickness

$$\frac{\partial H}{\partial t} = - \underbrace{\nabla \cdot \mathbf{Q}}_{\text{Flux divergence: deformation due to velocity gradients}} + a_s - a_b$$

Flux divergence:
deformation due
to velocity
gradients

where $\nabla \cdot \mathbf{Q} = \left(\frac{\partial \bar{\mathbf{u}}}{\partial x} + \frac{\partial \bar{\mathbf{v}}}{\partial y} \right) H$

A simple illustration in 1D:



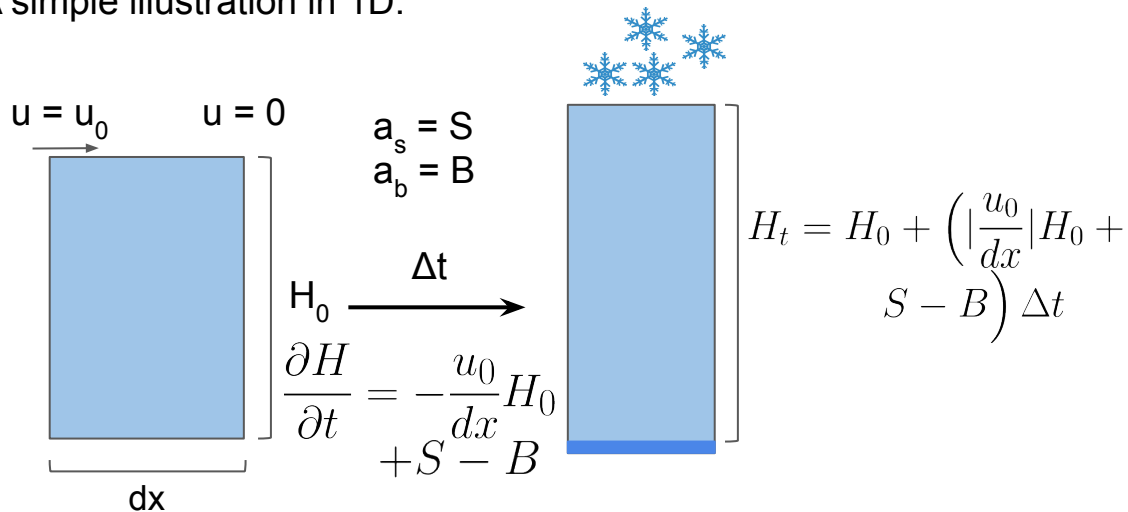
Mass conservation and evolution of ice thickness

$$\frac{\partial H}{\partial t} = - \underbrace{\nabla \cdot \mathbf{Q}}_{\text{Flux divergence: deformation due to velocity gradients}} + a_s - a_b$$

Flux divergence:
deformation due
to velocity
gradients

where $\nabla \cdot \mathbf{Q} = \left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \right) H$

A simple illustration in 1D:



Momentum conservation

$$\nabla \sigma + \rho \mathbf{g} = \mathbf{0}$$

All forces acting on a volume of ice must be balanced by forces acting on the sides.

$$\begin{aligned}\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + \rho g_x &= 0 \\ \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + \rho g_y &= 0 \\ \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z &= 0\end{aligned}$$

Writing the above relationship out for all directions yields three equations.

Subscripts indicate the direction of the stress and the direction normal to the surface the stress acts on. These are symmetric, so $\sigma_{xz} = \sigma_{zx}$, $\sigma_{yx} = \sigma_{xy}$, *etc.*

Stresses with two different subscripts (e.g., σ_{xz}) indicate *shear*, while two of the same (e.g., σ_{xx}) indicate compression or tension.

A flow law for glacier ice

$$\dot{\epsilon}_{ij} = A\tau^{n-1}\tau_{ij}$$

where n is usually taken to be 3

(Note: n is dependent on ice fabric (crystal orientations), and probably varies between about 1 and 4. We almost always assume $n=3$, given a general lack of constraints)

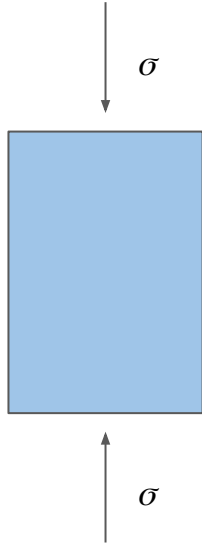
$\dot{\epsilon}_{ij}$ strain rate tensor describes deformation in all directions

τ_{ij} deviatoric stress tensor describes all stresses minus mean pressure

τ is the effective shear stress = $(\frac{1}{2} [\text{sum of squares of } \sigma_{ij}])^{1/2}$
(second invariant of σ_{ij})

A is a temperature-dependent rate factor

A simple illustration of ice rheology



Take the example of ice under vertical compression, with no other stresses acting on it.

The full stress tensor is:

$$\sigma_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma \end{pmatrix}$$

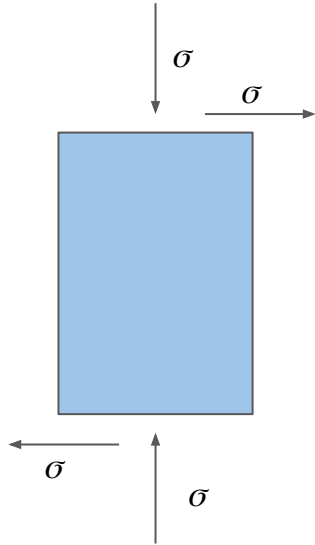
and the mean pressure $P = \sigma/3$

So the deviatoric stress tensor is:

$$\tau_{ij} = \begin{pmatrix} -\sigma/3 & 0 & 0 \\ 0 & -\sigma/3 & 0 \\ 0 & 0 & 2\sigma/3 \end{pmatrix} \longrightarrow \tau^2 = (2\sigma^2/9 + 4\sigma^2/9)/2 = \sigma^2/3$$

$$\text{So, } \dot{\epsilon}_{zz} = A \tau^2 \tau_{zz} = A \times \sigma^2/3 \times 2\sigma/3 = 2A\sigma^3/9$$

A simple illustration of ice rheology



Now let's add shear.

The full stress tensor is:

$$\sigma_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma \\ 0 & \sigma & \sigma \end{pmatrix}$$

and the mean pressure is still $P = \sigma/3$

So the deviatoric stress tensor is:

$$\tau_{ij} = \begin{pmatrix} -\sigma/3 & 0 & 0 \\ 0 & -\sigma/3 & \sigma \\ 0 & \sigma & 2\sigma/3 \end{pmatrix} \longrightarrow \tau^2 = (\sigma^2 + \sigma^2 + 2\sigma^2/9 + 4\sigma^2/9)/2 = 4\sigma^2/3$$

So, $\dot{\epsilon}_{zz} = A \tau^2 \tau_{zz} = A \times 4\sigma^2/3 \times 2\sigma/3 = 8A\sigma^3/9$ (compared with $2A\sigma^3/9$ when no shear)

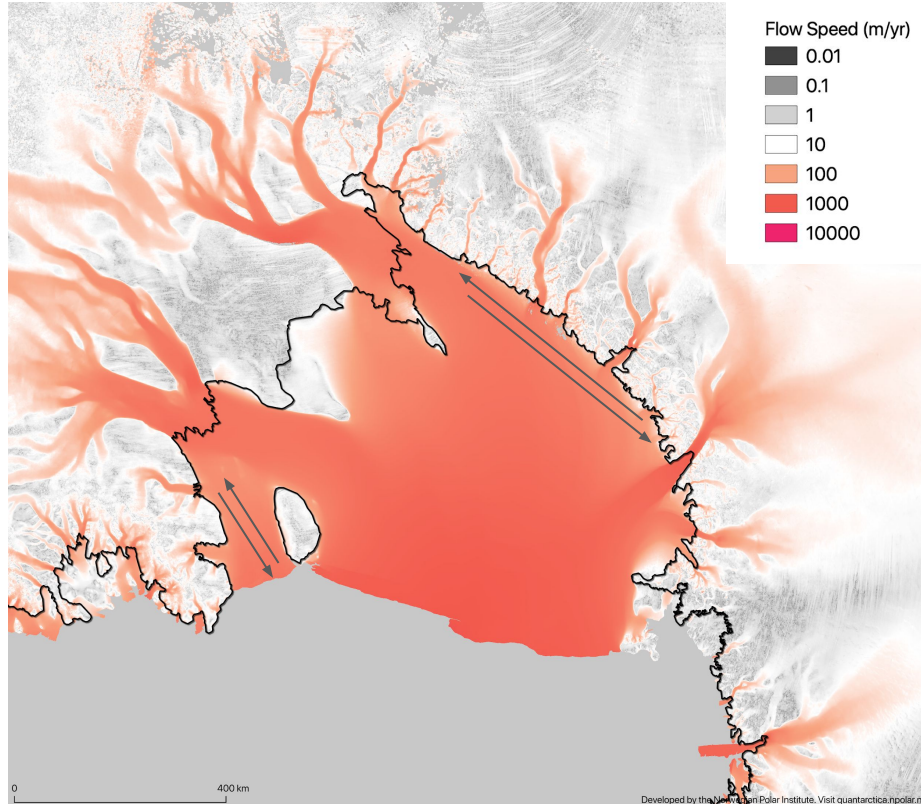
That's a lot of math, but the takeaway is that adding shear caused 4x faster deformation in the vertical direction compared with compression alone.

→ Ice is shear-softening!

A recipe for calculating velocities from stresses

1. Determine components of the stress tensor; subtract off mean stress to get deviatoric stress tensor τ_{ij}
2. Calculate the effective shear stress $\tau^2 = \frac{1}{2}$ [sum of squares of elements of τ_{ij}]
3. Now we have all the components to calculate strain rates from flow law (assuming we know A or ice temperature)
4. Integrate strain rates to get ice velocities
5. Apply boundary conditions

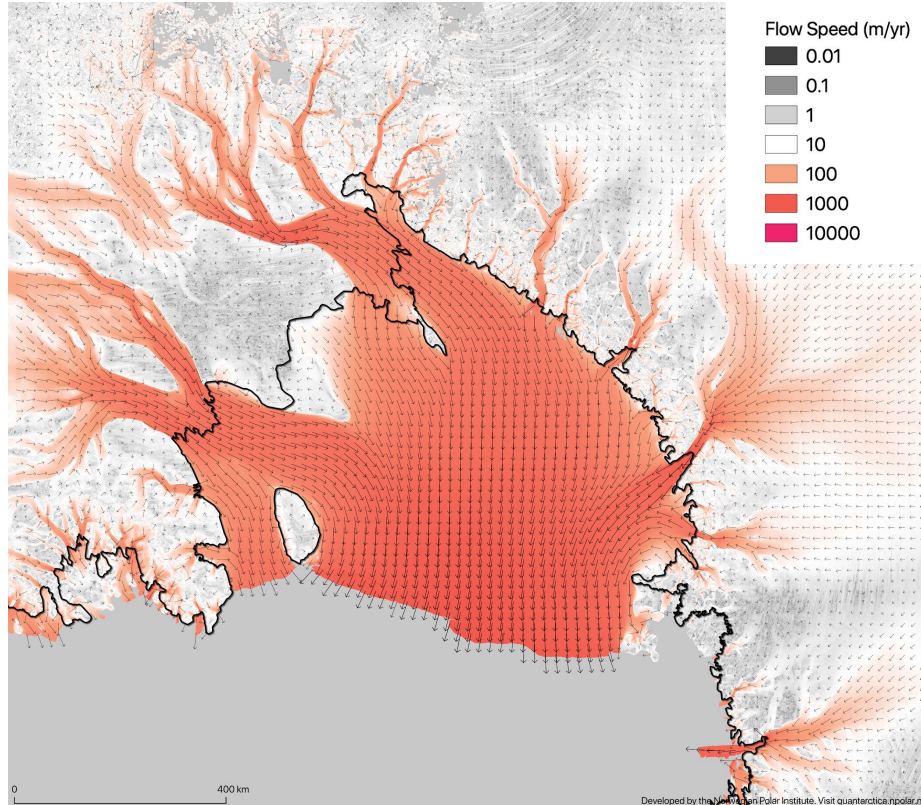
Ice Shelf Buttressing



For grounded ice in contact with the ocean, there is a force imbalance at the grounding line, which leads to an extensional stress.

Shear stress along embayment walls resists ice shelf flow. Stresses transfer through ice shelves with essentially zero lag (imagine pushing down one side of a floating sheet of ice; it will tip rather than deform). So this resistance is transferred all the way to the grounding line.

Ice rises' role in buttressing

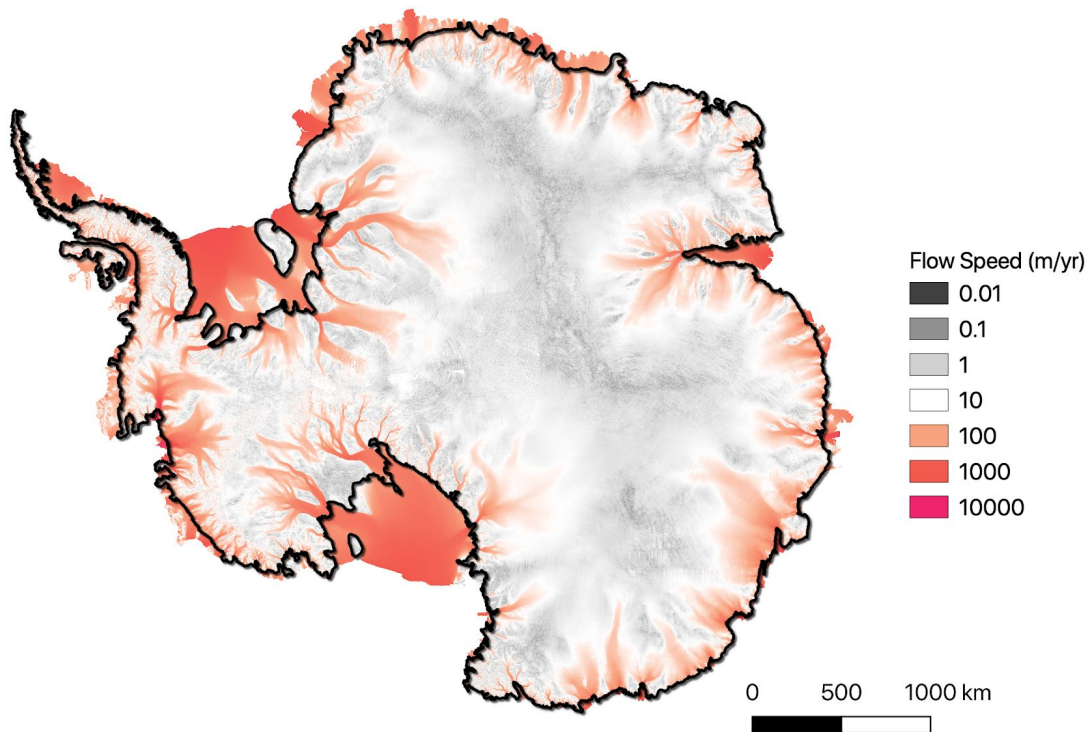


Ice rises often divert ice shelf flow to either side.

This cause convergent flow, which thickens the ice shelf.

Ice shelves tend to only be stable between such lateral pinning points.

Flow regimes in ice sheets



Ice sheet interiors: vertical shear

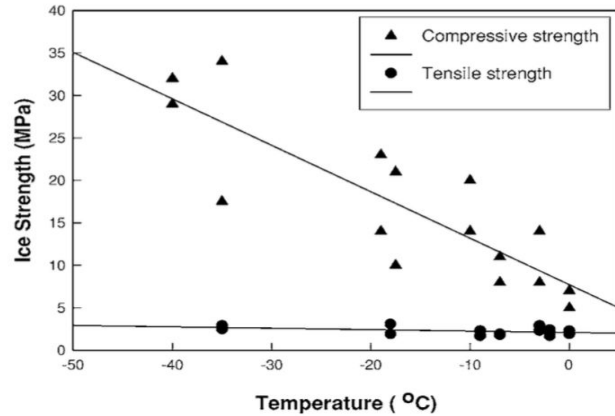
Ice streams: longitudinal stretching with horizontal shear at sides

Ice shelves: longitudinal stretching with horizontal shear at sides (but the sides are often much more distant)

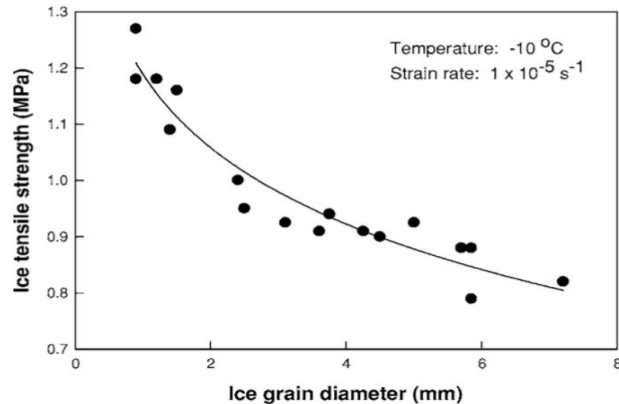
Valley glaciers: vertical shear, horizontal shear at walls, longitudinal stretching where bed is lubricated.

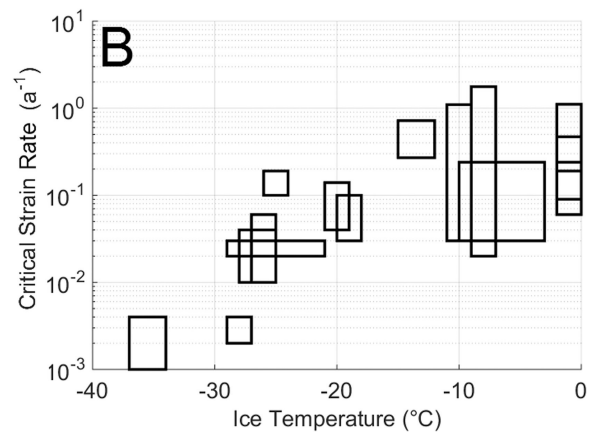
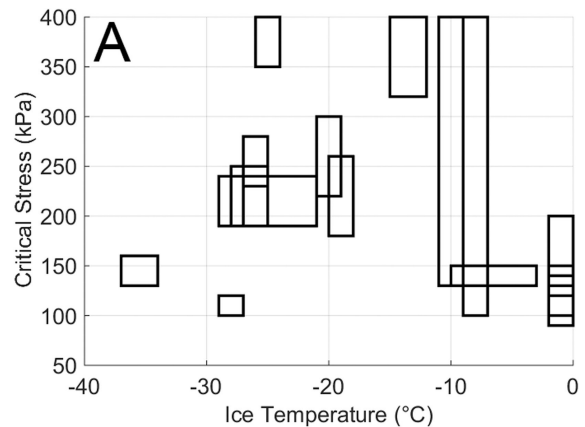
Ice rises: Vertical shear, longitudinal stretching and/or shear at grounding line

Fracture and crevasse formation

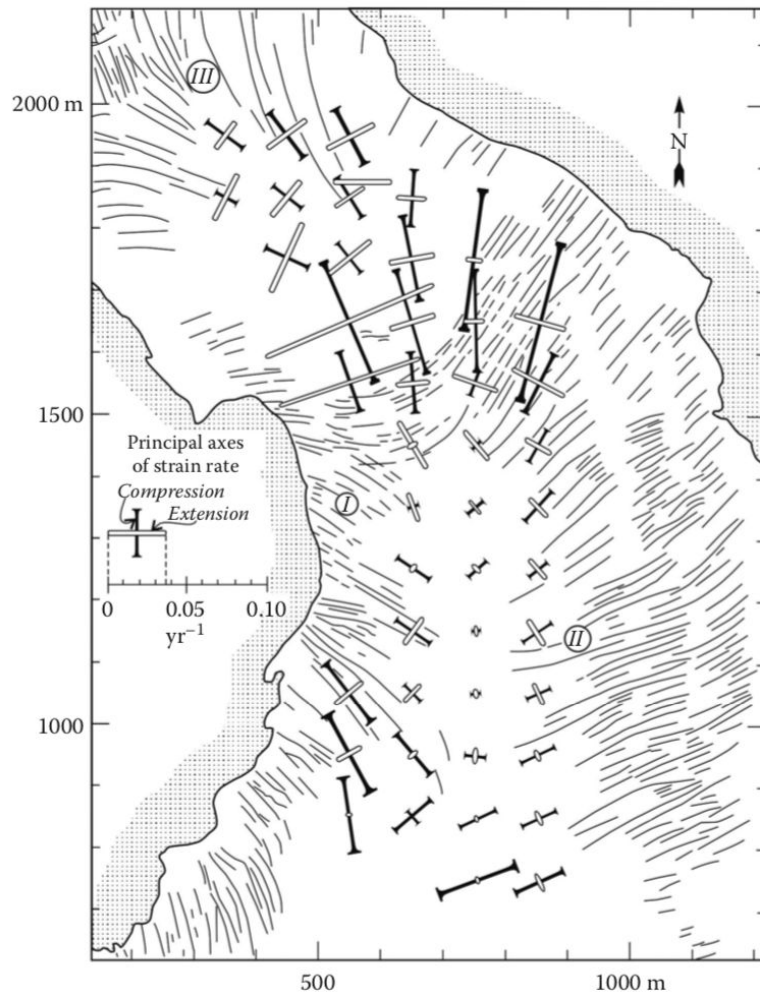


- Ice breaks easily under tension, but not under compression
- Tensile strength relatively insensitive to temperature
- Strong dependence of tensile strength on grain size



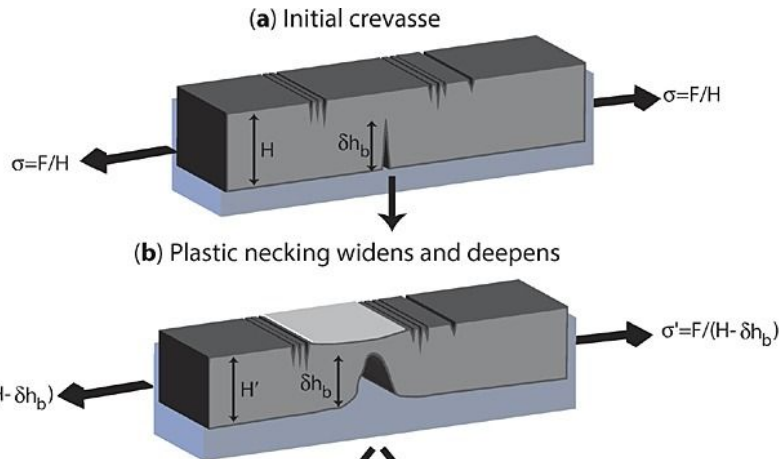


Colgan et al. (2016). Glacier crevasses:
Observations, models, and mass balance
implications

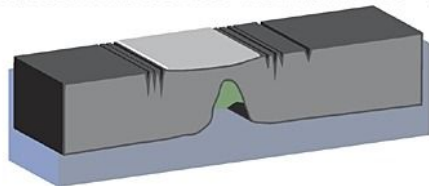


Crevassing and ice-shelf dynamics

Crevasse initiating melting or freezing:

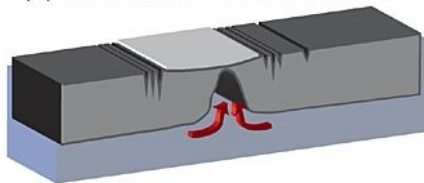


(c) Cold ocean fills crevasse with marine ice



Bassis & Ma (2015): Evolution of basal crevasses links ice shelf stability to ocean forcing

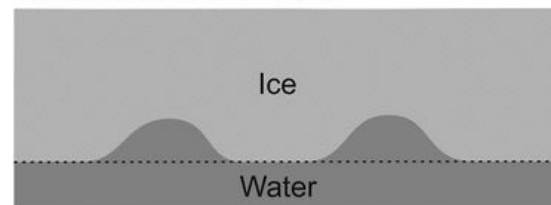
(d) Warm ocean erodes crevasse



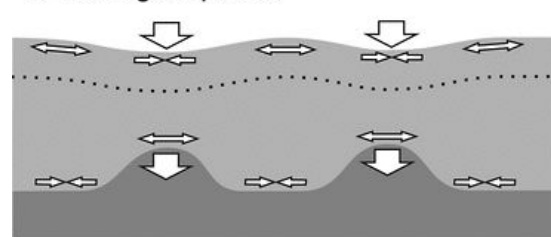
Vaughan et al. (2012): Subglacial melt channels and fracture in the floating part of Pine Island Glacier, Antarctica

Melting initiating crevassing:

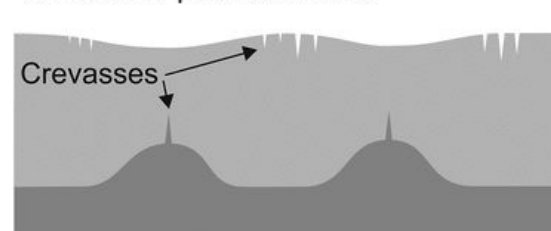
a. Undeformed ice shelf

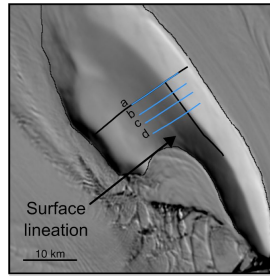
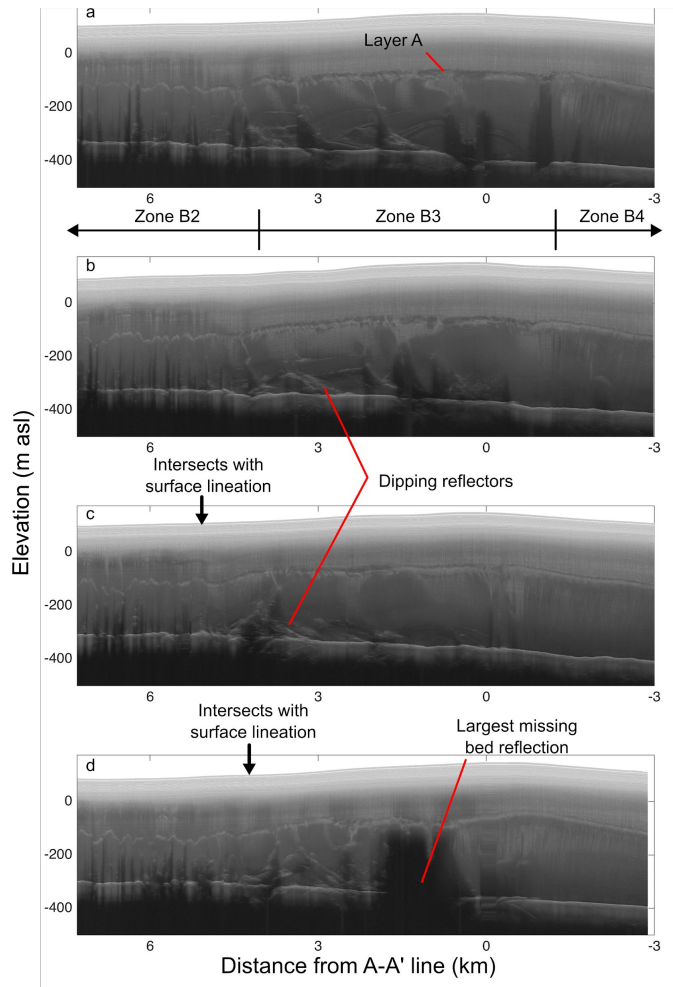


b. Flexing response



c. Zones of possible failure



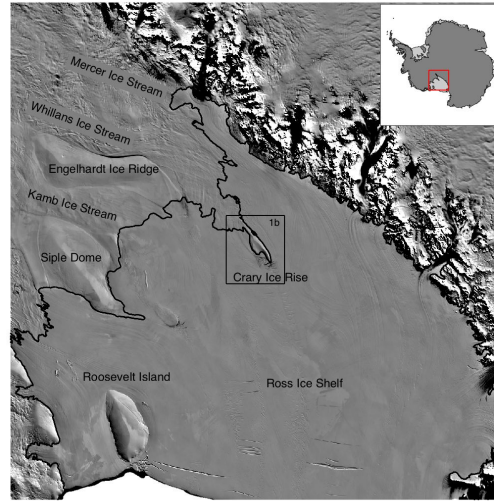


Shameless plug for my own research

Crary Ice Rise used to be part of the Ross Ice Shelf, until it settled on a rebounding bed ~1 kyr ago.

Radar transects across Crary Ice Rise show large areas of missing bed reflection, as well as lots of interesting structures.

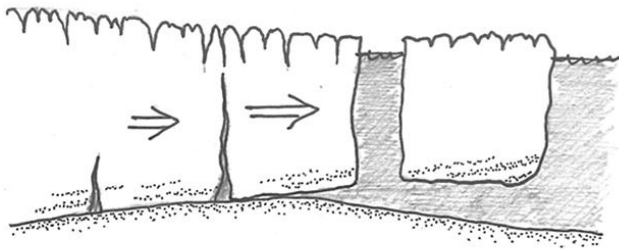
The most likely explanation for these is that they are marine ice-filled fractures in the former ice shelf.



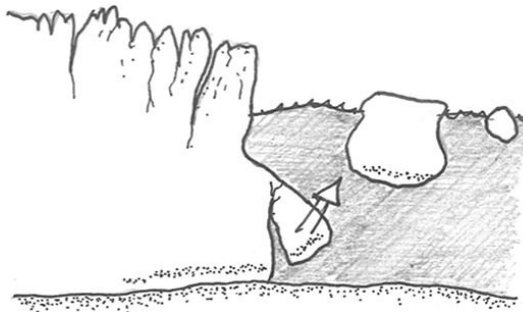
Hillebrand et al. (*in review*). Radio-echo sounding of Crary Ice Rise, Antarctica reveals abundant marine ice in former ice shelf rifts and basal crevasses

Calving

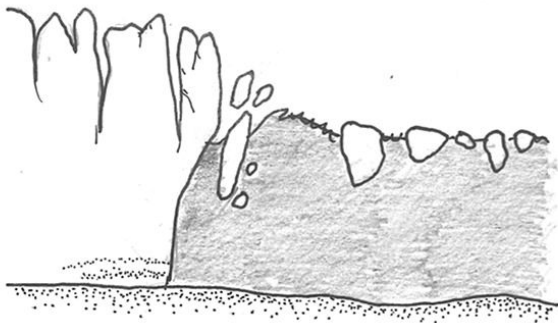
A: Longitudinal extension



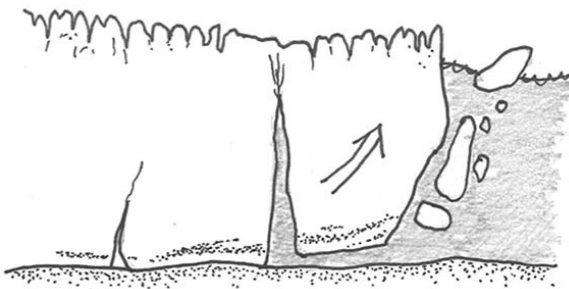
C: Buoyant calving - ice foot



B: Melt-undercutting



D: Buoyant calving - full thickness



Calving depends on:

- Strain rates at calving front
- Pre-existing fractures (both open crevasses and small cracks)
- Presence of water in crevasses
- Non-uniform ice front geometry
- Elastic and brittle processes

So it's basically impossible to model.

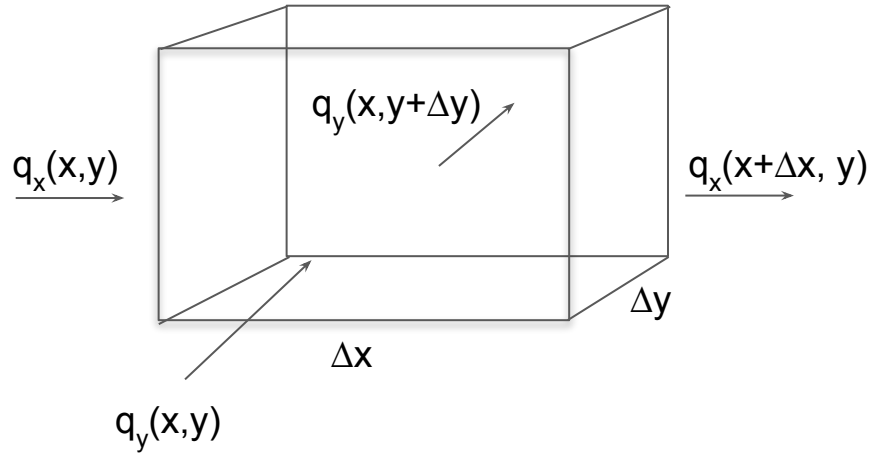
Most models use a parameterization based on stresses or strain rates, and then tune to match observations.

Summary

- Ice deforms under its own weight, driven by surface slope (grounded ice) or thickness gradients (floating ice)
- We use a non-linear flow law to describe the deformation of ice in response to applied stresses.
 - Strain rate \propto (deviatoric stress)³
 - Ice is a shear-softening material
- Using the flow law and conservation laws, we can calculate velocities and ice thickness changes due to applied stresses.
- Ice shelves and ice rises resist flow across the grounding line, which helps to stabilize marine ice sheets.
- Calving and fracturing of ice are very hard to predict.

tl;dr: Ice just wants to be flat. It finds interesting ways to do that. Sometimes it breaks.

Mass conservation derivation



Let's imagine part of a glacier, with a fixed volume. If this volume stays the same, then:
Flux entering = Flux leaving

$$\text{Flux entering} = q_x(x, y)\Delta y + q_y(x, y)\Delta x$$

$$\text{Flux leaving} = q_x(x + \Delta x, y)\Delta y + q_y(x, y + \Delta y)\Delta x$$

$$q_x(x, y)\Delta y + q_y(x, y)\Delta x = q_x(x + \Delta x, y)\Delta y + q_y(x, y + \Delta y)\Delta x$$

$$q_x(x + \Delta x, y)\Delta y - q_x(x, y)\Delta y + q_y(x, y + \Delta y)\Delta x - q_y(x, y)\Delta x = 0$$

$$(q_x(x + \Delta x, y)\Delta y\Delta x - q_x(x, y)\Delta y\Delta x)/\Delta x + (q_y(x, y + \Delta y)\Delta x\Delta y - q_y(x, y)\Delta x\Delta y)/\Delta y = 0$$

Definition of derivative:

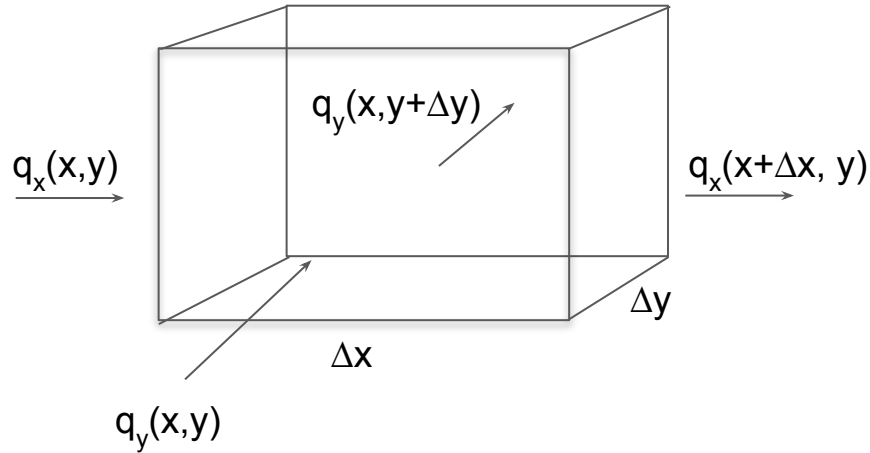
$$\lim(\Delta x \rightarrow 0) [q_x(x + \Delta x, y) - q_x(x, y)]/\Delta x = \partial q_x / \partial x$$

$$\lim(\Delta y \rightarrow 0) [q_y(x, y + \Delta y) - q_y(x, y)]/\Delta y = \partial q_y / \partial y$$

$$\partial q_x / \partial x \Delta x \Delta y + \partial q_y / \partial y \Delta x \Delta y = 0$$

$$\partial q_x / \partial x + \partial q_y / \partial y = 0$$

Mass conservation derivation



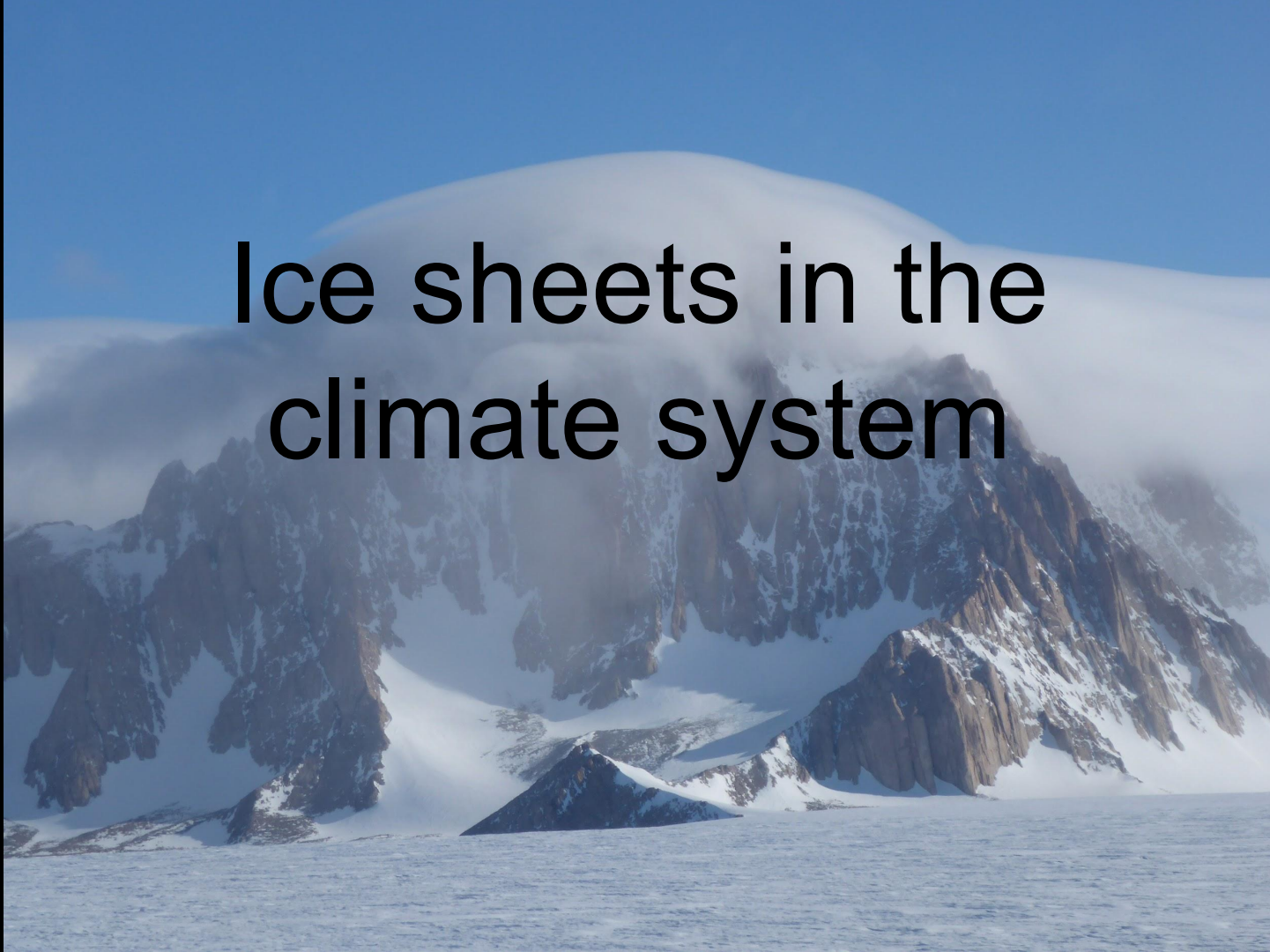
If not in steady state, then:
Flux in - Flux out = $\partial H / \partial t \Delta x \Delta y$

$$\rightarrow \partial H / \partial t + \partial q_x / \partial x + \partial q_y / \partial y = 0$$

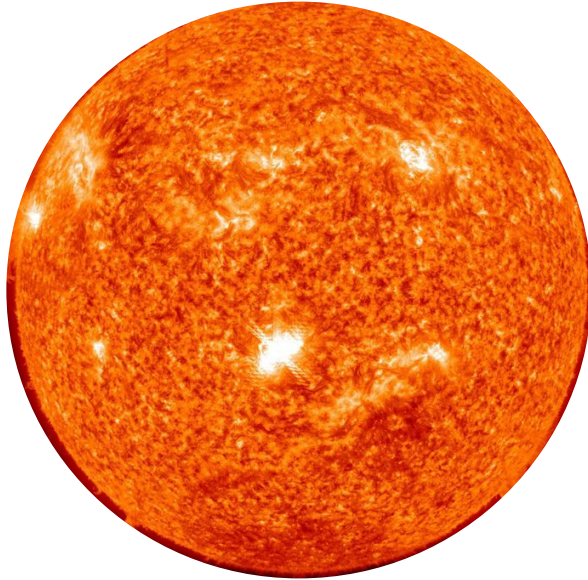
If we account for mass entering and leaving through the top (a_s) and bottom (a_b) (snowfall, melt, sublimation, freeze-on):

$$\partial H / \partial t + \partial q_x / \partial x + \partial q_y / \partial y = a_s + a_b$$

Ice sheets in the climate system



Global energy budget



$$\begin{aligned}\text{Power in} &= (\text{Solar energy flux}) \times (\text{fraction energy absorbed}) \times (\text{area}) \\ &= S (1 - \alpha) \pi R_E^2\end{aligned}$$

$$\begin{aligned}\text{Power out} &= (\text{surface area}) \times (\text{emissivity}) \times (\text{temperature})^4 \times (\text{constant}) \\ &= 4 \pi R_E^2 \varepsilon \sigma T^4\end{aligned}$$

In steady state (i.e., no global temperature change):

$$\text{Power in} = \text{Power out}$$

$$S (1 - \alpha) = 4 \varepsilon \sigma T^4$$

$$\text{where } S = 1361 \text{ W m}^{-2}$$

$$\alpha = \sim 0.32$$

$$\varepsilon = \sim 1$$

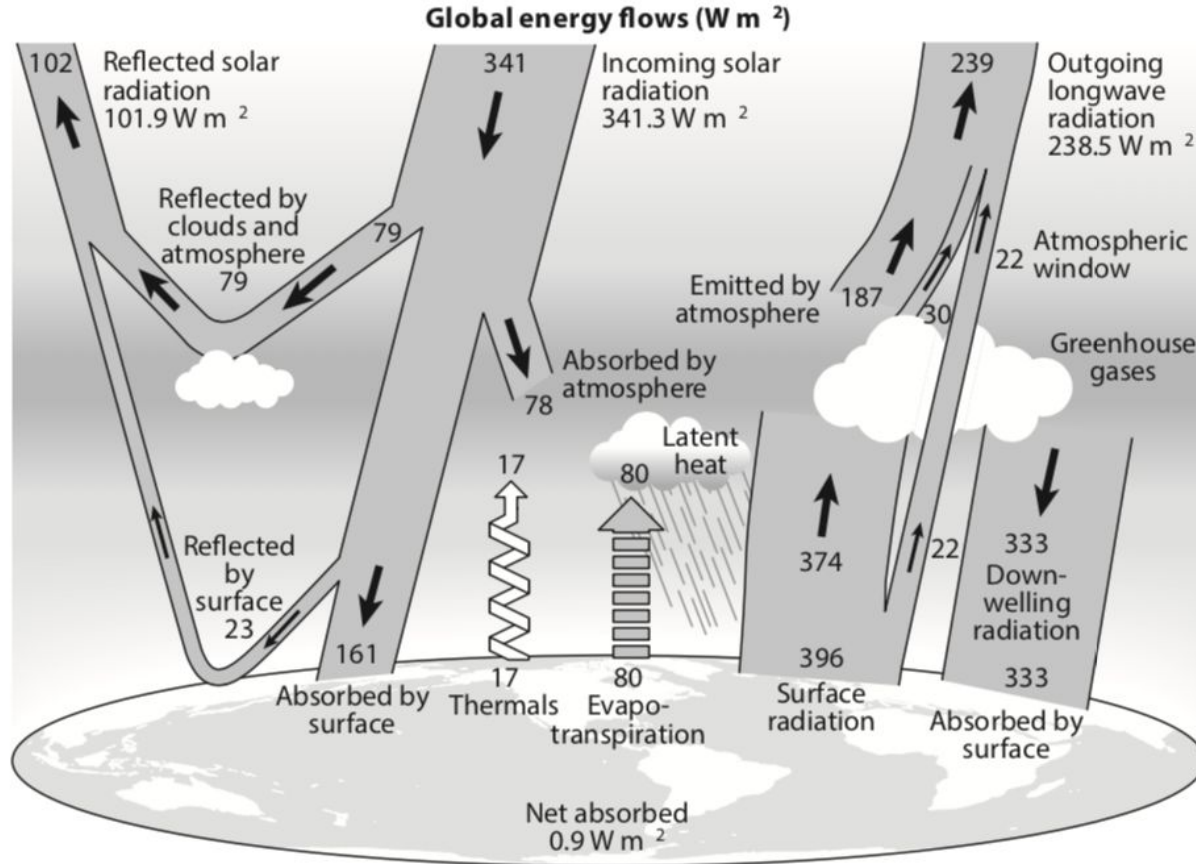
$$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

Using this relationship, solve for the steady-state temperature of the Earth.

253 K (-20°C). That seems too cold.



Global energy budget

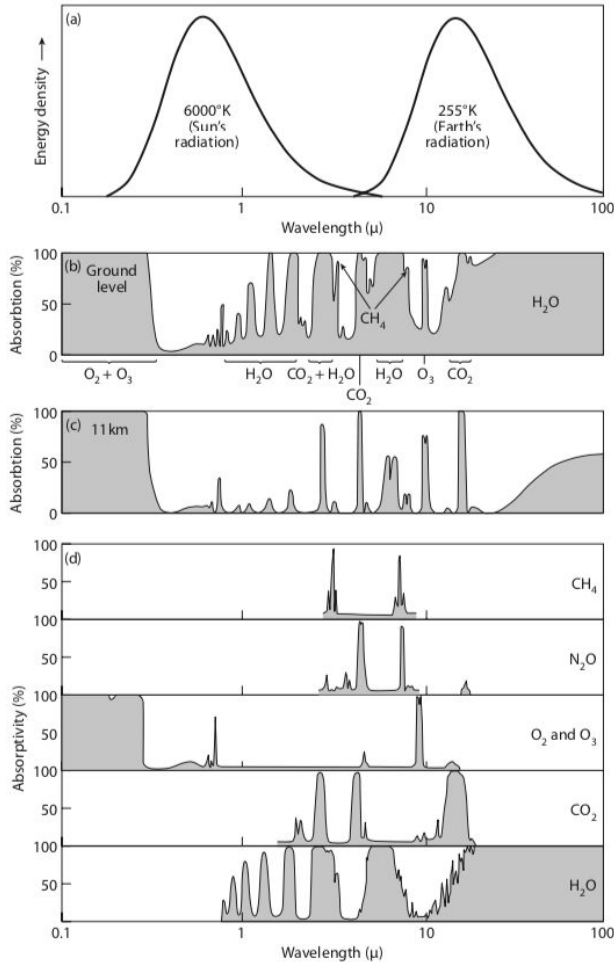


The Greenhouse Effect:
Earth absorbs visible radiation (sunlight) and emits infrared radiation.

Greenhouse gases absorb infrared radiation emitted by the Earth.

Greenhouse gases emit infrared radiation evenly to space and to the Earth.

Haigh and Cargill (2015). The Sun's Influence on Climate



The Greenhouse Effect:
Earth absorbs visible radiation (sunlight) and emits infrared radiation.

Greenhouse gases absorb infrared radiation emitted by the Earth.

Greenhouse gases emit infrared radiation evenly to space and to the Earth.

This increases the average surface temperature of the earth from -20°C to a balmy 14°C .

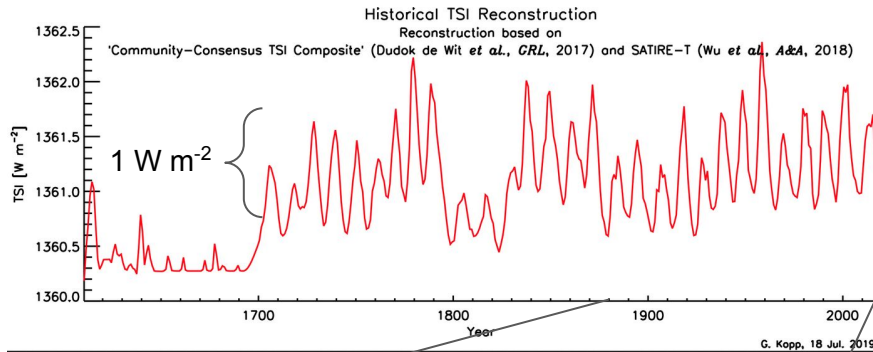
Bender (2013): Paleoclimate

Radiative forcing: solar vs. greenhouse effects

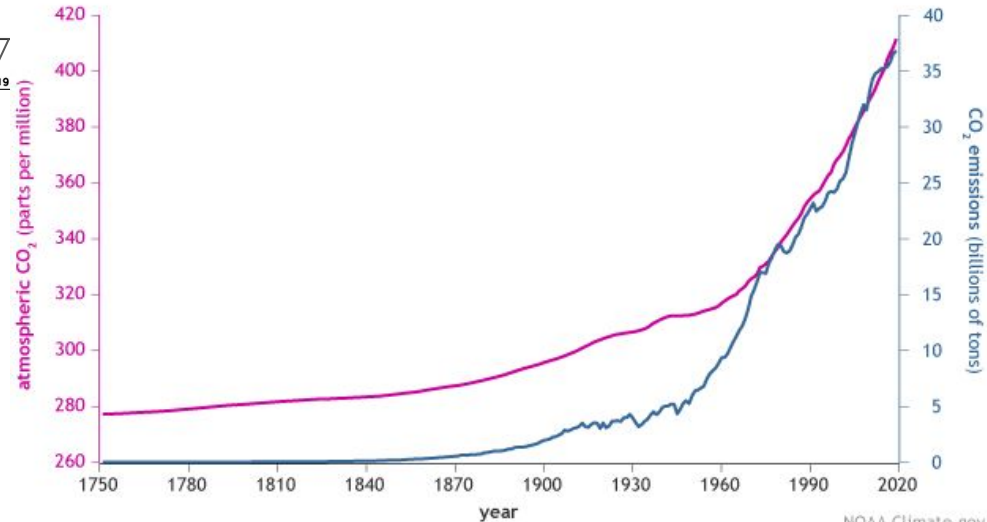
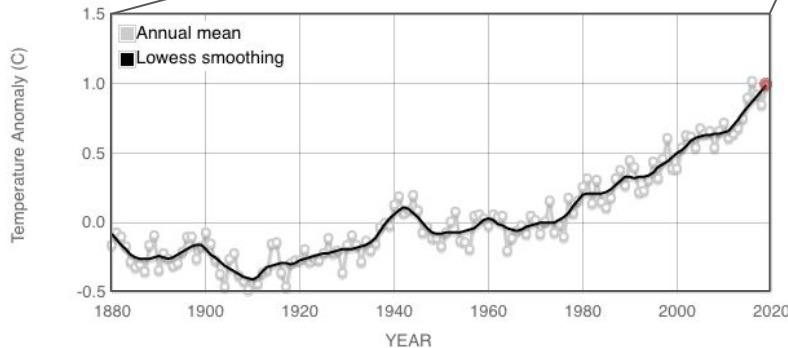
$$\Delta \text{Forcing} = \Delta S (1 - \alpha) / 4 \rightarrow$$

$$1 \text{ W m}^{-2} (1 - 0.32) / 4 = 0.17 \text{ W m}^{-2}$$

Doubling $\text{CO}_2 = 4 \text{ W m}^{-2}$ or $\sim 2.7 \text{ W m}^{-2}$ from human activity since 1750



CO_2 in the atmosphere and annual emissions (1750-2019)



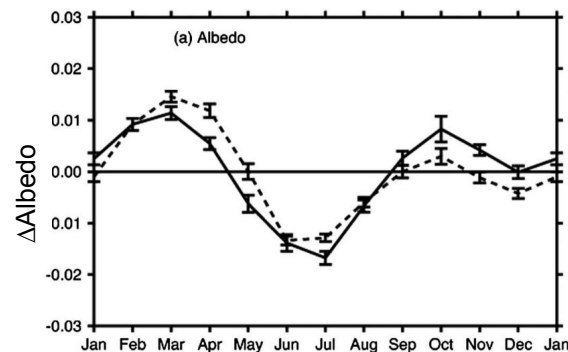
Activity: Experiments with a simple global energy balance model

Background: [Simple Climate Models](#) and [Simple Climate Models cont'd](#)

Using the [0-D Energy Balance Model](#):

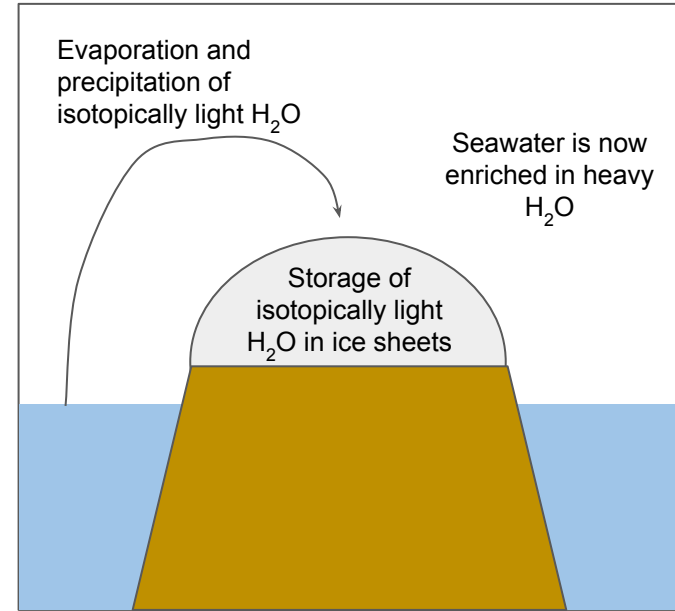
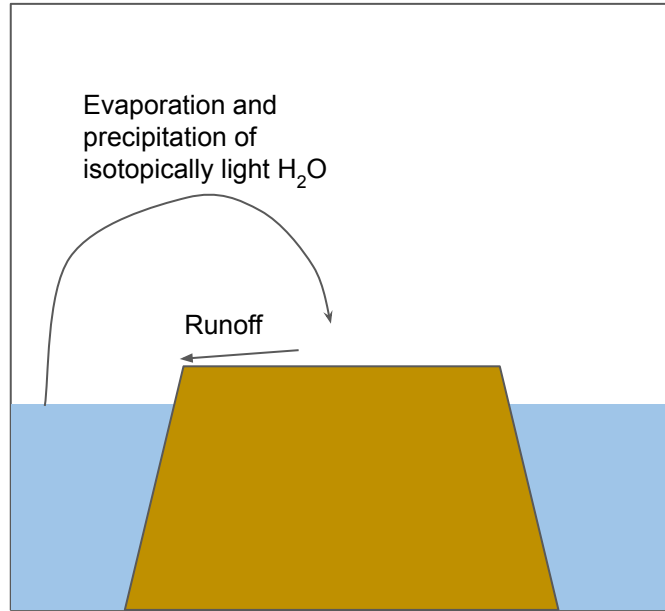
1. Compute temperature sensitivity to changes in Solar Constant ($K/(W\ m^{-2})$) for black- and grey-body (mid-range IPCC) Earth
 - Keep CO_2 levels at 280 ppmv (pre-industrial); albedo at 0.32
 - How much temperature change results from the range of total solar irradiance from prior slide ($1361 \pm 1\ W\ m^{-2}$). How do measured temperature changes of 1–1.5 K since 1880 compare with your results?
2. Compute temperature sensitivity to CO_2 ($K/ppmv\ CO_2$) for black- and grey-body (mid-range IPCC) Earth
 - Keep Solar Constant = $1361\ W\ m^{-2}$; albedo at 0.32
 - Using data from previous slide: How do measured temperature changes compare with your results?
3. Compute the range of global albedo possible for the Last Glacial Maximum
 - Grey body (mid-range) IPCC Earth
 - $CO_2 = 185\ ppmv$ during a glacial period
 - Solar Constant = $1361\ W\ m^{-2}$
 - global temperatures $4 \pm 1^\circ C$ cooler than pre-industrial
 - How does this compare with the seasonal cycle of albedo at the present day (shown right)? What might account for similarities and differences?

NB: Enter numbers into the boxes instead of using sliders.
Sometimes moving one slider resets another value.



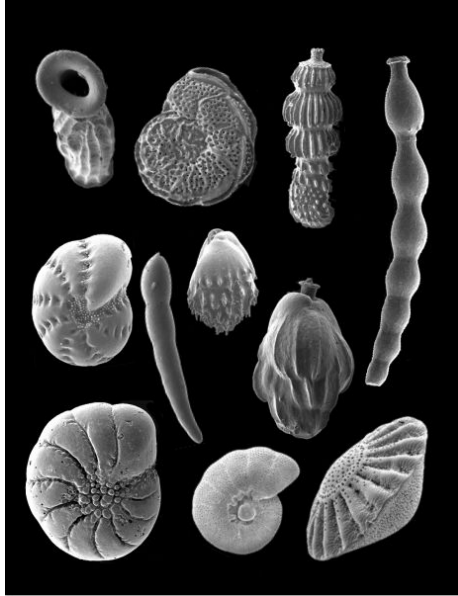
[Stephens et al. \(2015\). The albedo of Earth](#)

Records: benthic oxygen isotopes

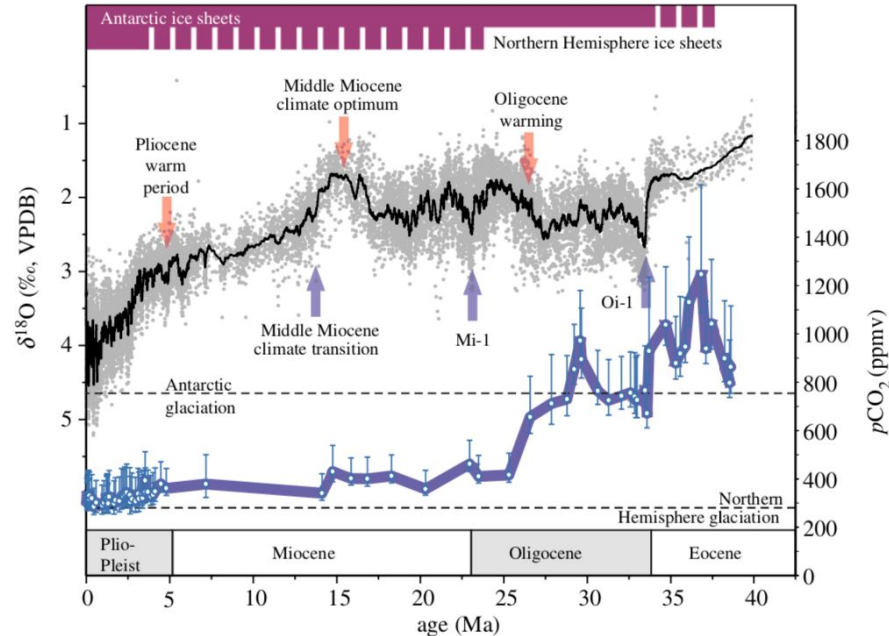


Isotopic signature of seawater gets recorded in benthic foraminifera.

Records: benthic oxygen isotopes



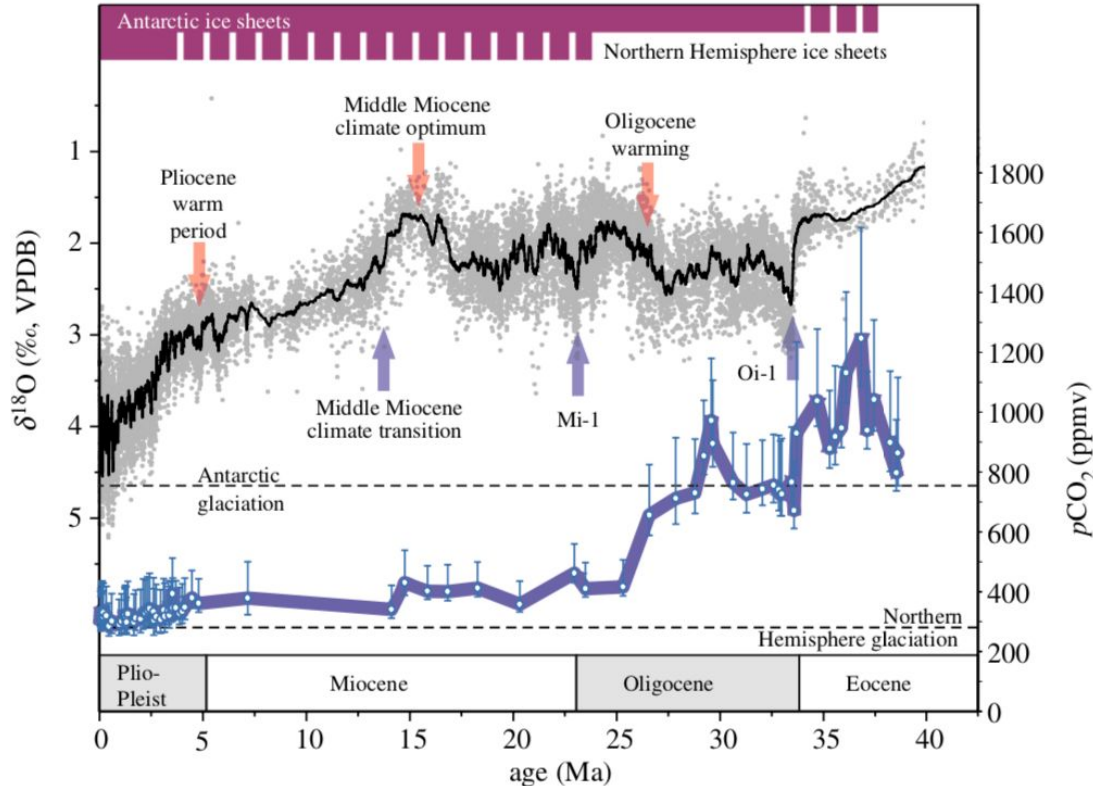
Pearson 2012: Oxygen isotopes in foraminifera: Overview and historical review



Zhang YG, Pagani M, Liu Z, Bohaty SM, DeConto R. 2013. A 40-million-year history of atmospheric CO₂

lower $\delta^{18}\text{O}$ → isotopically lighter seawater → warmer, less ice
higher $\delta^{18}\text{O}$ → isotopically heavy seawater → colder, more ice

Trends in Cenozoic climate



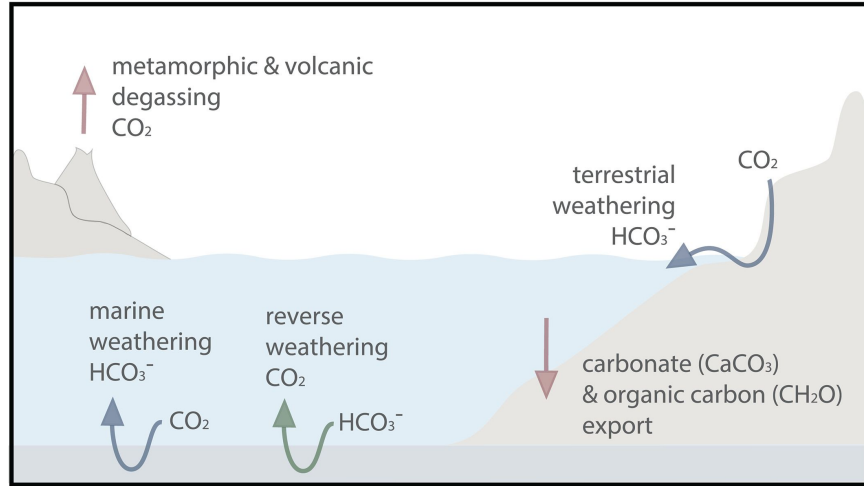
Progressive lowering of $p\text{CO}_2$ and global mean temperatures since 40 Ma.

Onset of permanent Antarctic glaciation at the Eocene-Oligocene transition, as $p\text{CO}_2$ declined below $\sim 3\times$ pre-industrial values.

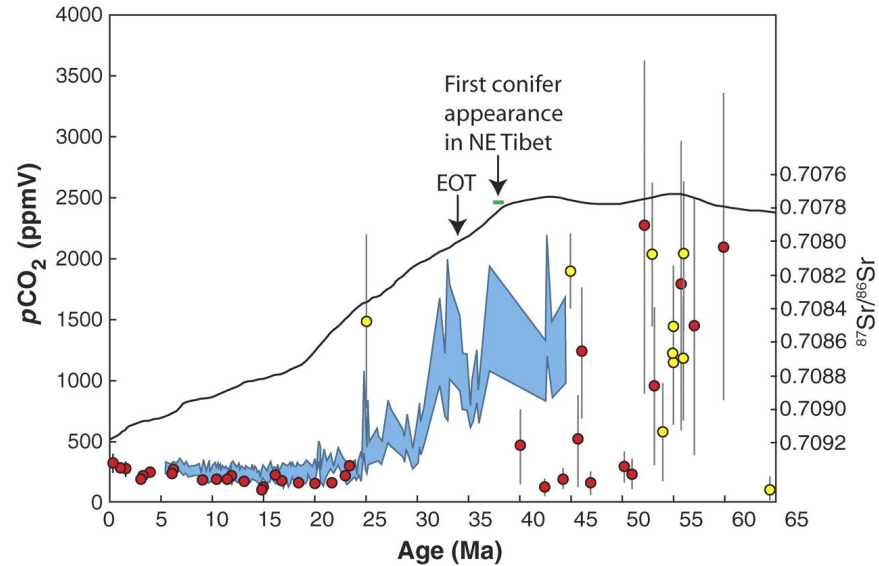
Northern Hemisphere glaciation greatly intensified ~ 3 Ma.

Much lower $p\text{CO}_2$ threshold for NH glaciation than for Antarctic, likely due to lower latitudes of land masses.

Possible drivers of trends in Cenozoic climate: increased weathering from the uplift of Tibet



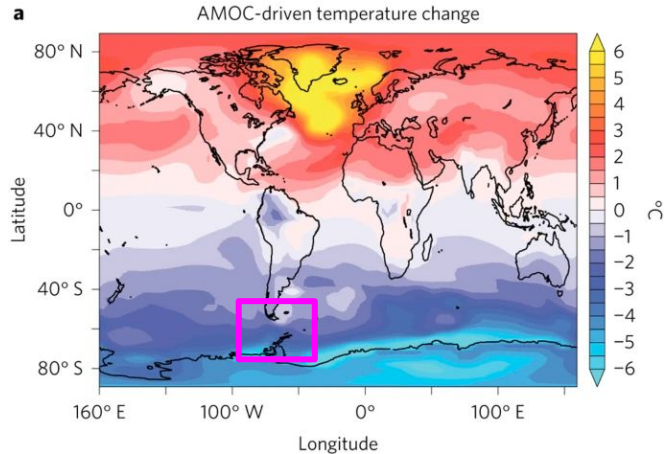
Isson et al. (2019): Evolution of the Global Carbon Cycle and Climate Regulation on Earth



Garziona (2008): Surface uplift of Tibet and Cenozoic global cooling

Increasing $^{87}\text{Sr}/^{86}\text{Sr}$ (note inverted scale) suggests increase in continental weathering flux to oceans coincident with rise of the Tibetan Plateau.

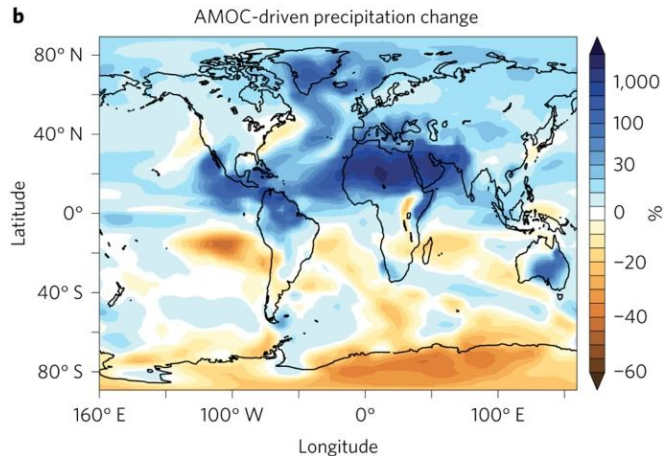
Possible drivers of trends in Cenozoic climate: opening and deepening of the Drake Passage



Simulated global climate with shallow (300 m) and deep (1500 m) Drake Passage

Deepened Drake Passage leads to strong decrease in Southern Hemisphere temperature and precipitation.

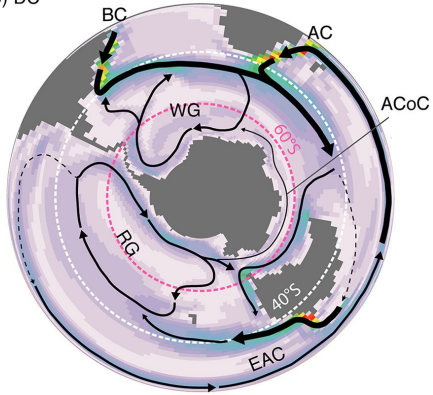
Increased NH precipitation and temperatures could have driven an increase in silicate weathering rates, causing drawdown in atmospheric $p\text{CO}_2$.



Elsworth et al. (2017): Enhanced weathering and CO_2 drawdown caused by latest Eocene strengthening of the Atlantic meridional overturning circulation

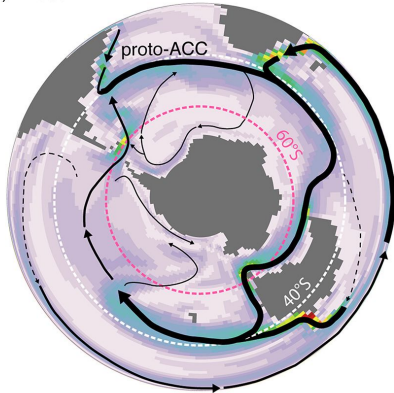
Possible drivers of trends in Cenozoic climate: opening and deepening of the Drake Passage

(b) DC *



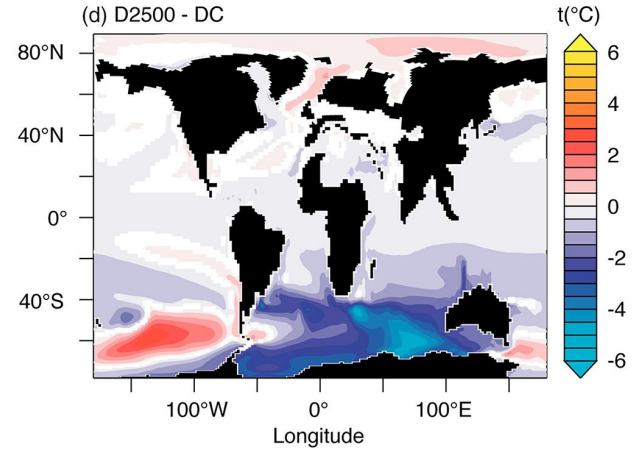
Drake Passage closed

(d) D2500 *



Drake Passage open,
2500 m deep

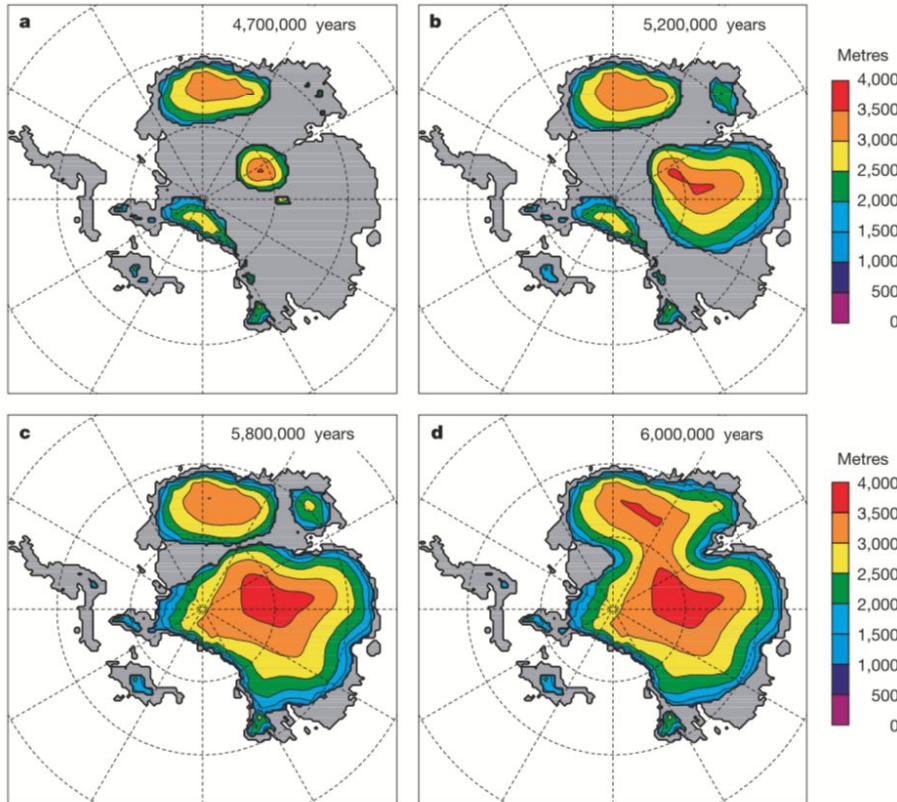
* Qualitative interpretation



Sea-surface temperature difference
(Drake open - Drake closed)

Toumoulin et al. (2020): Quantifying the Effect of the Drake Passage Opening on the Eocene Ocean

Possible drivers of trends in Cenozoic climate: ice-albedo feedback



Reduced CO₂ from 4x to 2x pre-industrial levels over 10 million years across the Eocene-Oligocene transition

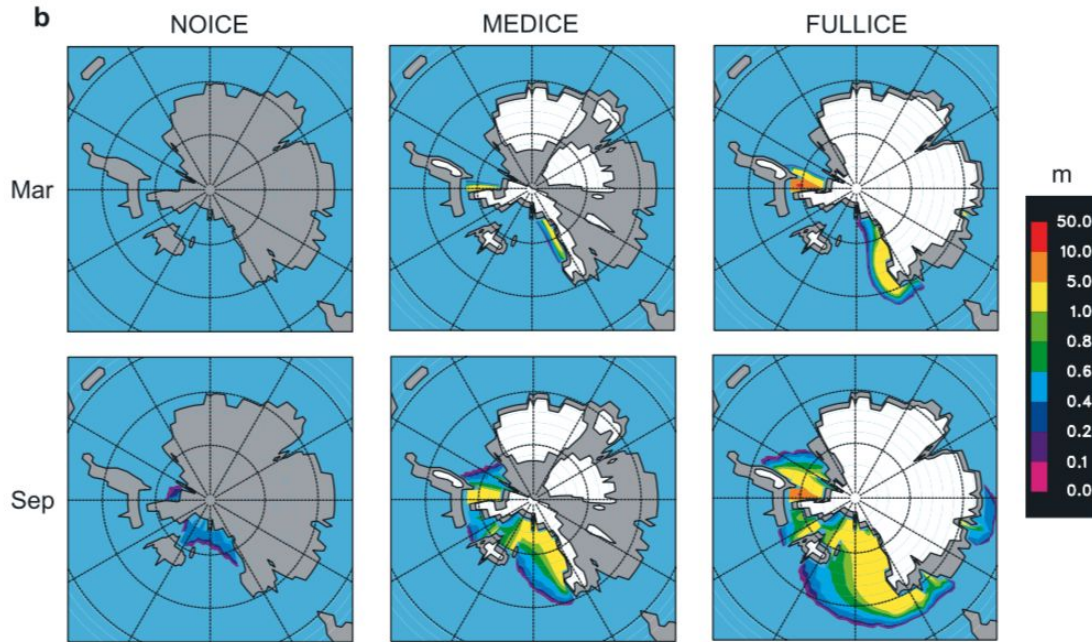
Initially, small ice-caps grow on high mountain ranges.

At 3x pre-industrial CO₂, ice caps begin to rapidly expand towards continental glaciation

Large leaps in ice sheet volume as independent ice caps coalesce.

DeConto and Pollard (2003): Rapid Cenozoic Glaciation of Antarctica Induced by Declining Atmospheric CO₂

Possible drivers of trends in Cenozoic climate: ice-albedo feedback



Reduced CO_2 from 4x to 2x pre-industrial levels over 10 million years across the Eocene-Oligocene transition

Initially, small ice-caps grow on high mountain ranges.

At 3x pre-industrial CO_2 , ice caps begin to rapidly expand towards continental glaciation

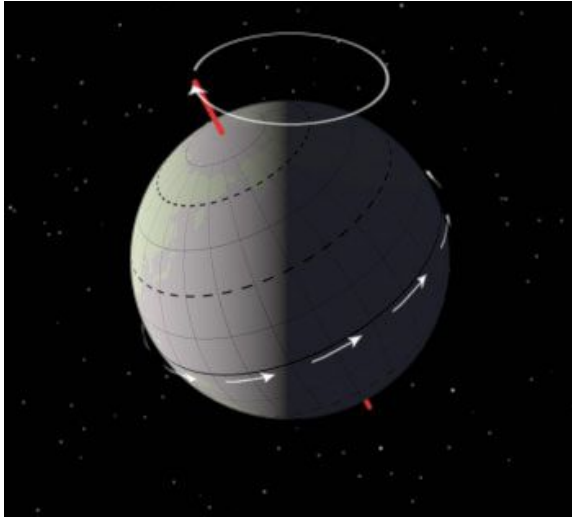
Large leaps in ice sheet volume as independent ice caps coalesce.

Ice sheets required for sea-ice growth.
Sea ice further increases albedo.

DeConto et al. (2007): Sea ice feedback and Cenozoic evolution of Antarctic climate and ice sheets

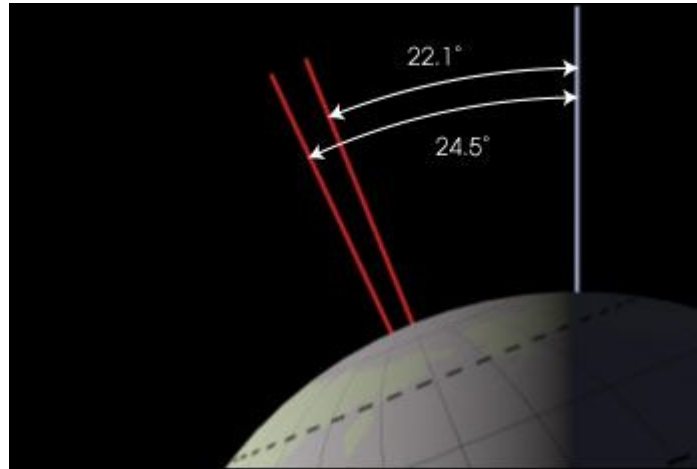
Milankovitch cycles

Precession: 19 kyr



Controls timing of perihelion (sun closest) and aphelion (sun furthest), and so modulates the effect of obliquity

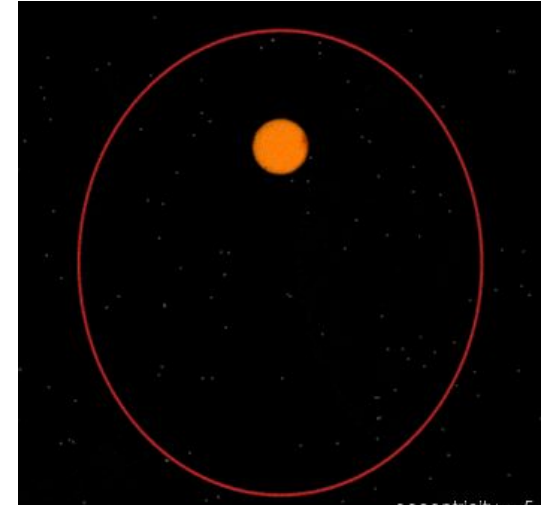
Obliquity: 41 kyr



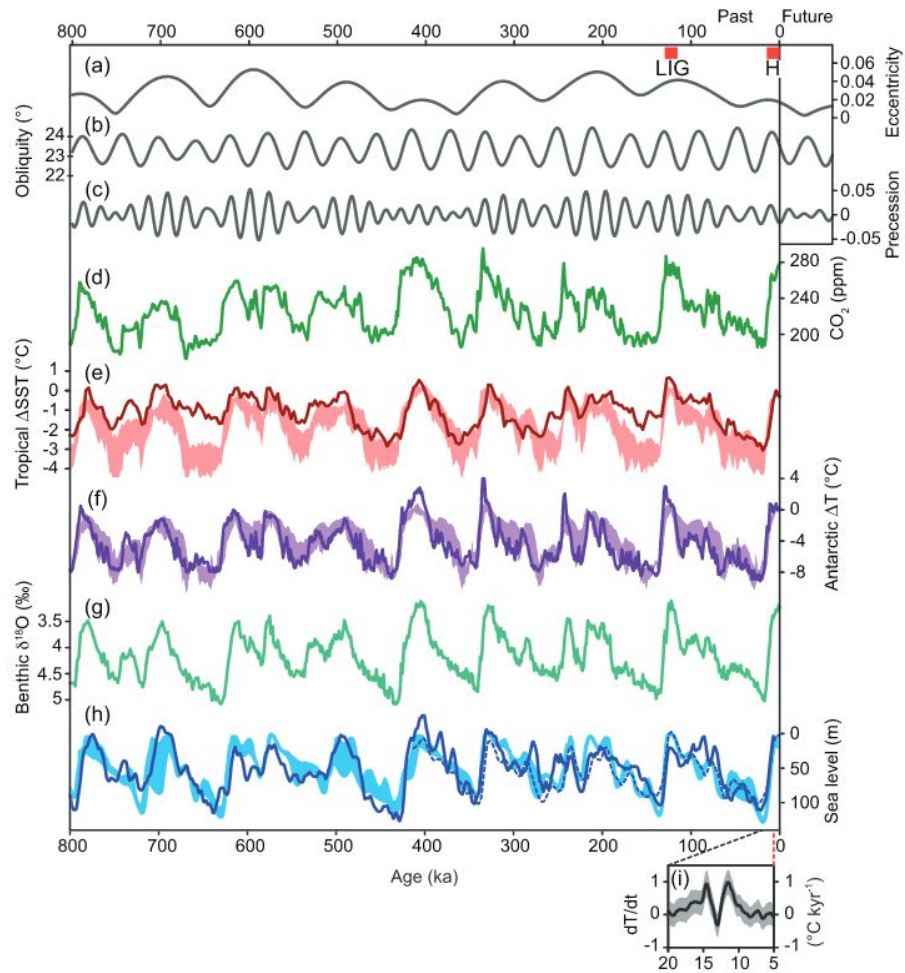
Increased tilt leads to higher seasonality at high latitudes.
Little effect at low latitudes

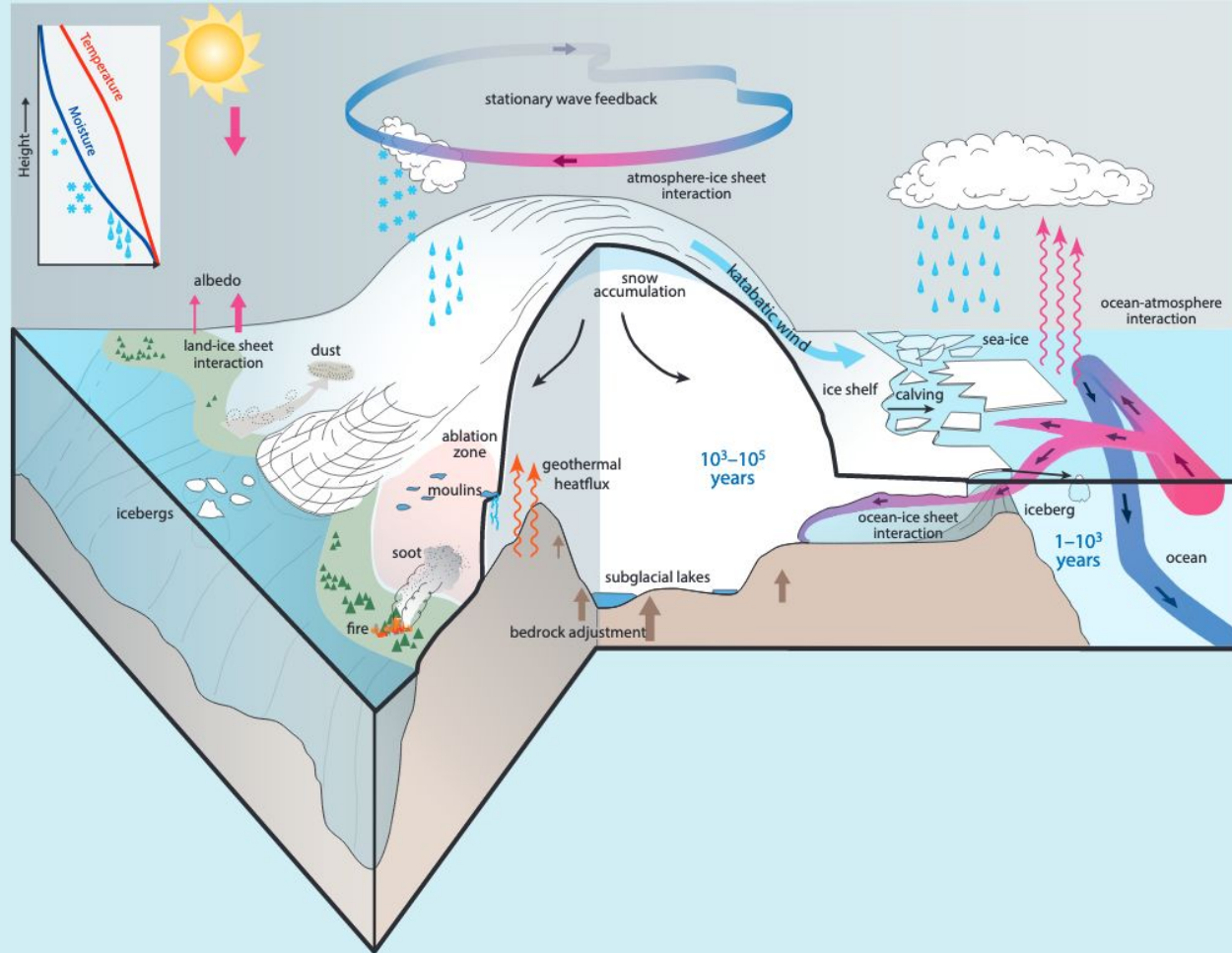
https://www.earthobservatory.nasa.gov/features/Milankovitch/milankovitch_2.php

Eccentricity: 100 kyr



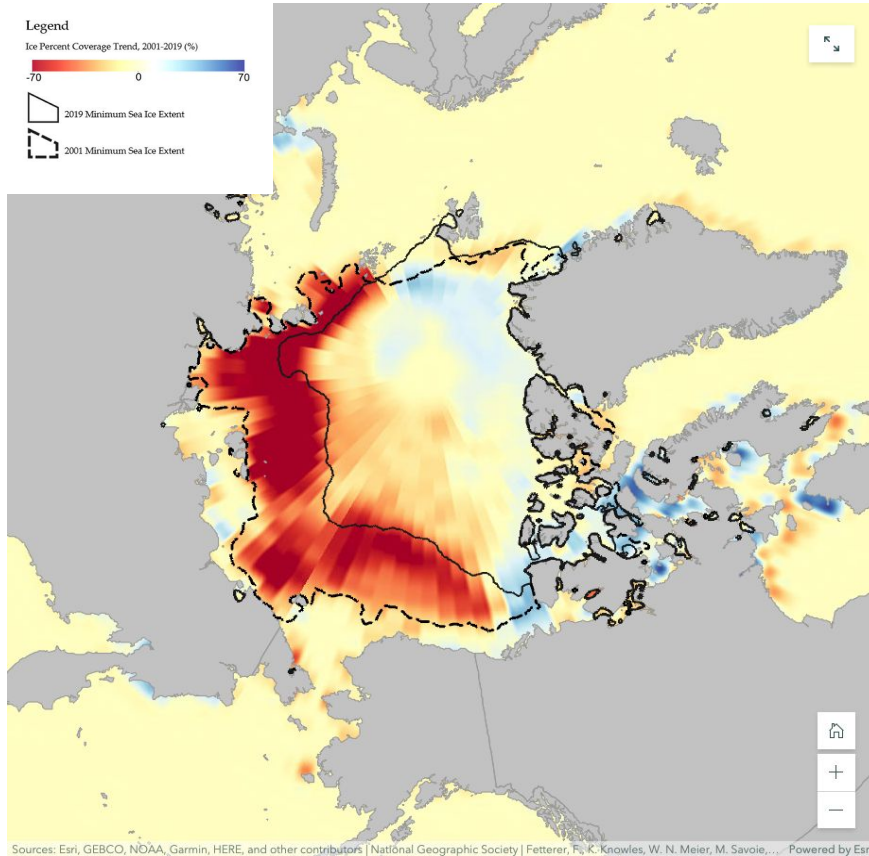
Smallest effect on insolation



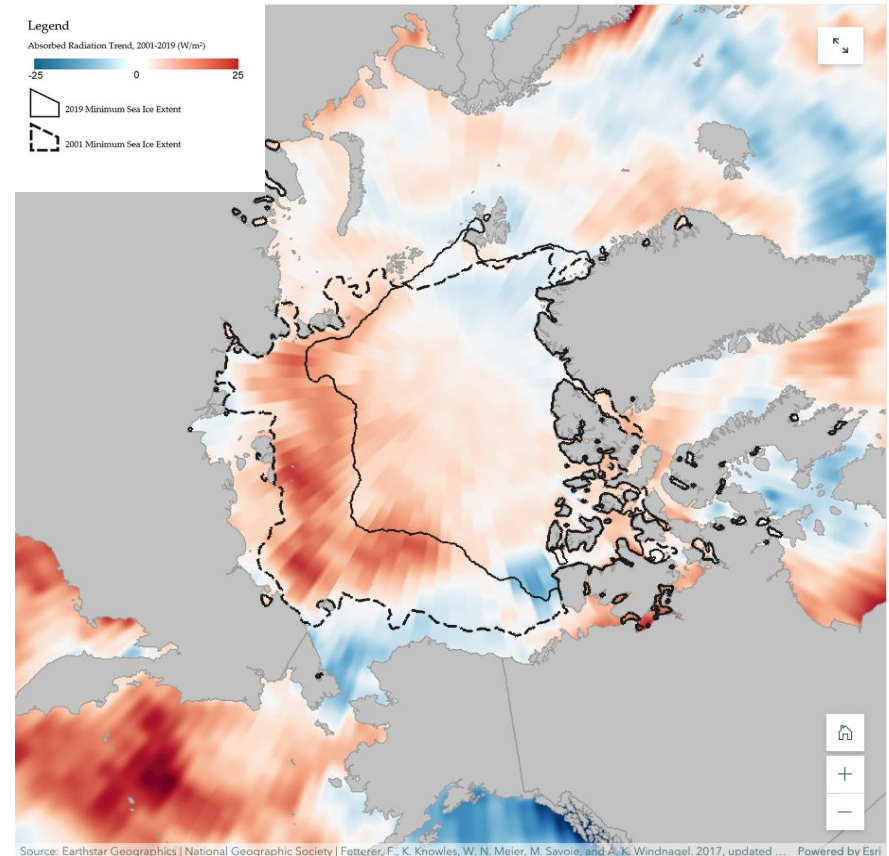


Box 5.2, Figure 1 | Schematic illustration of multiple interactions between ice sheets, solid earth and the climate system which can drive internal variability and affect the coupled ice sheet–climate response to external forcings on time scales of months to millions of years. The inlay figure represents a typical height profile of atmospheric temperature and moisture in the troposphere.

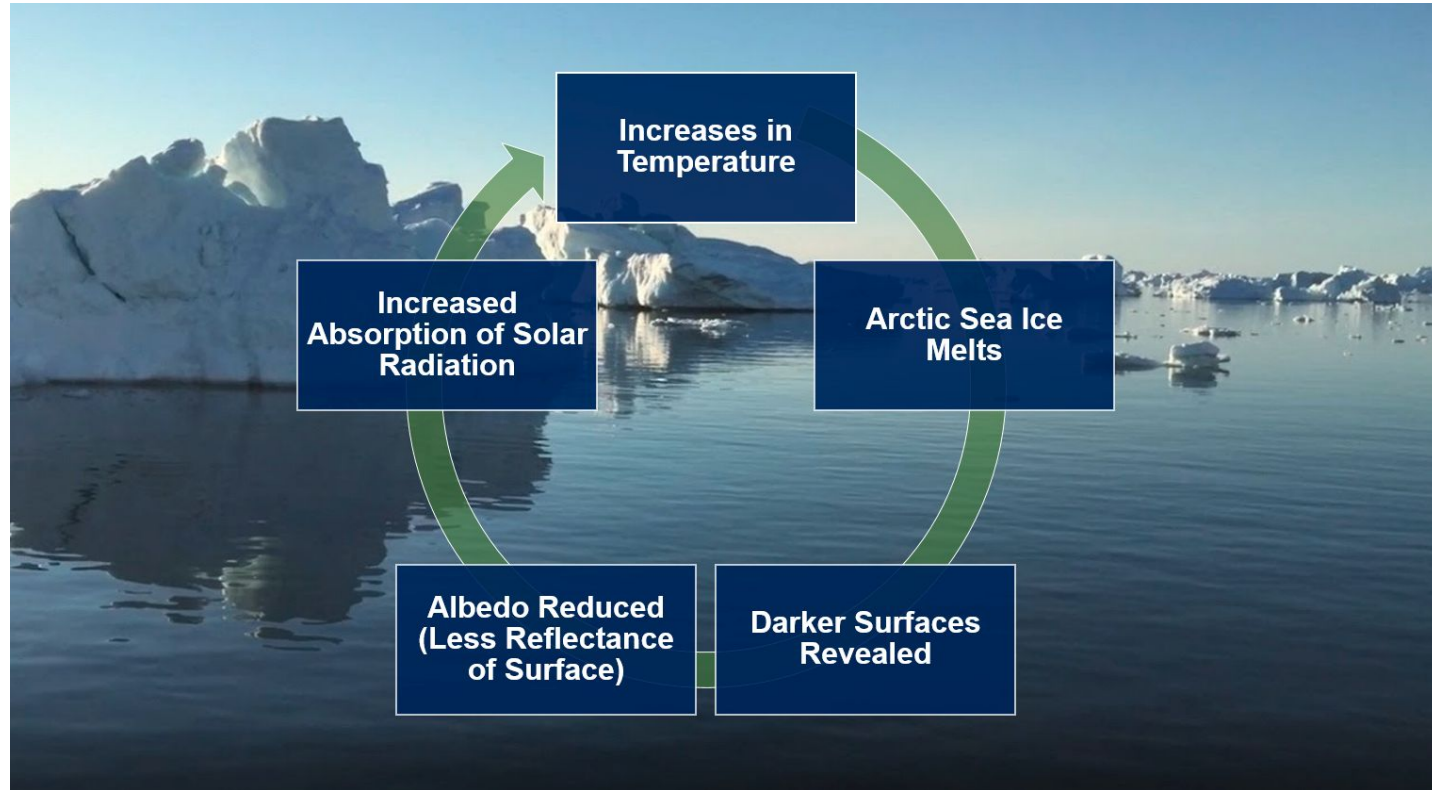
Polar amplification



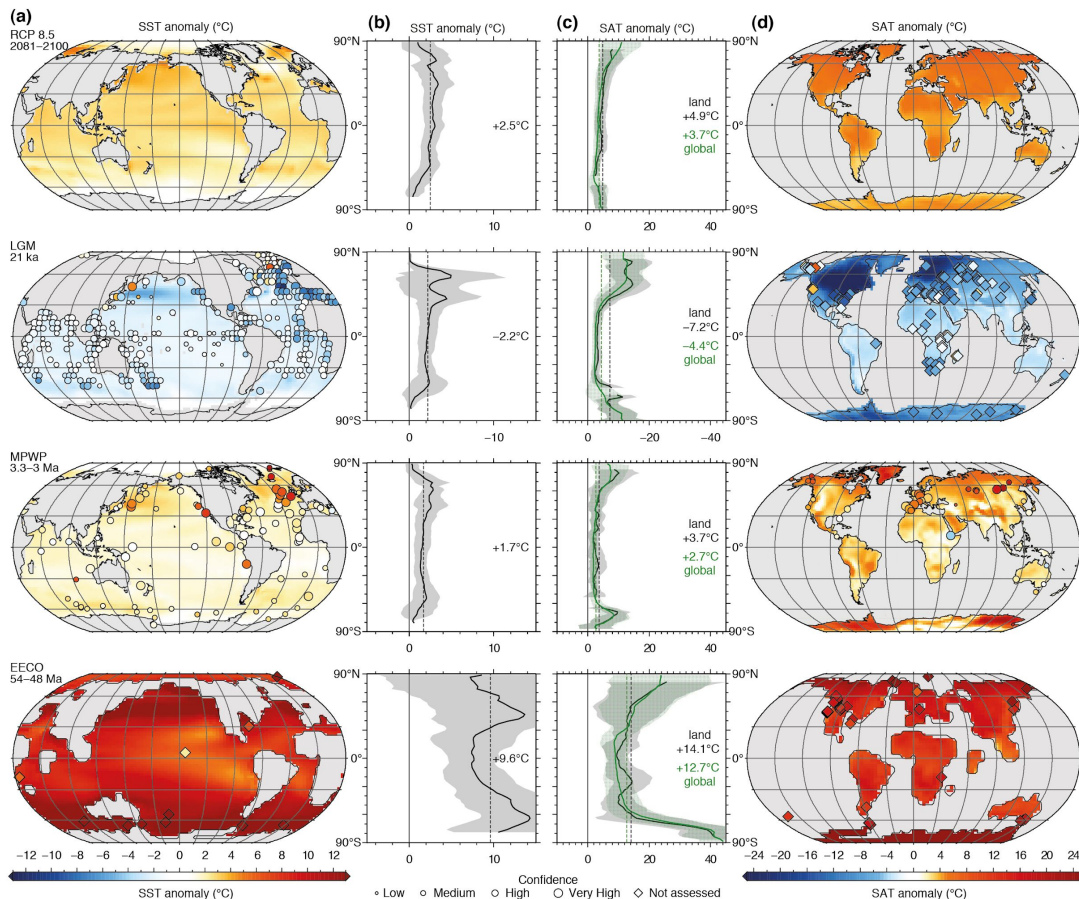
Source: NASA story-map on [ice-albedo feedbacks in the Arctic](#)



Polar amplification



Polar amplification



The poles tend to warm or cool more than the rest of the planet.

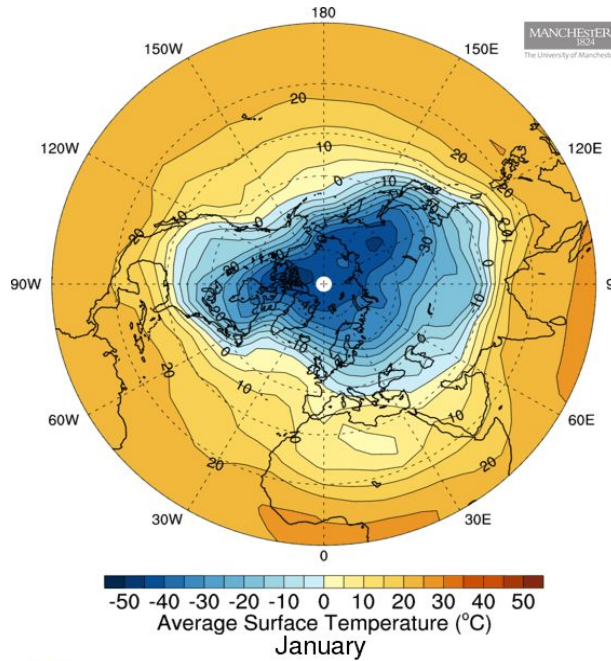
This is primarily driven by the sea ice-albedo feedback.

Over the timescale of glacial cycles, orography also contributes because of the considerable height of ice sheets.

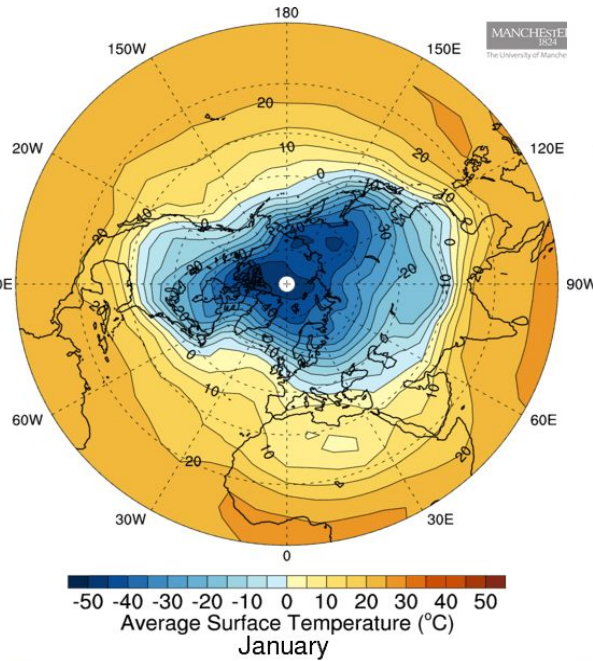
Polar amplification is present across many (all?) climate states.

Partially explains why ice sheets are sensitive to relatively small changes in CO₂ and global mean temperature.

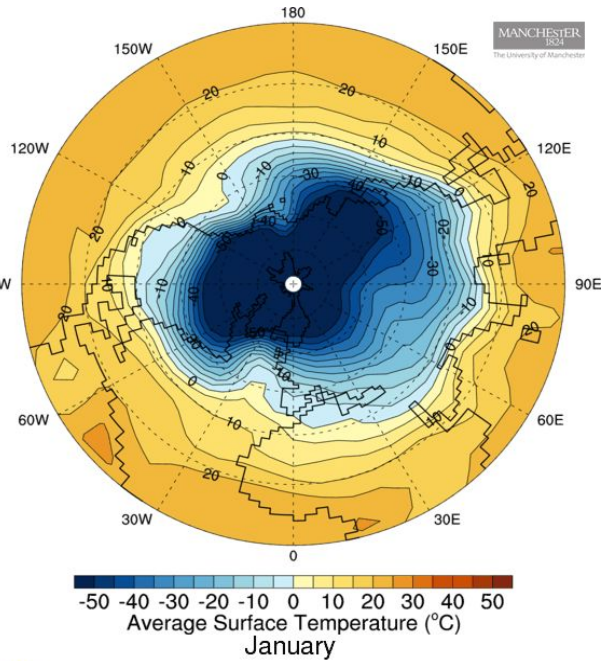
Polar amplification



Pre-industrial CO₂ and
orbital parameters

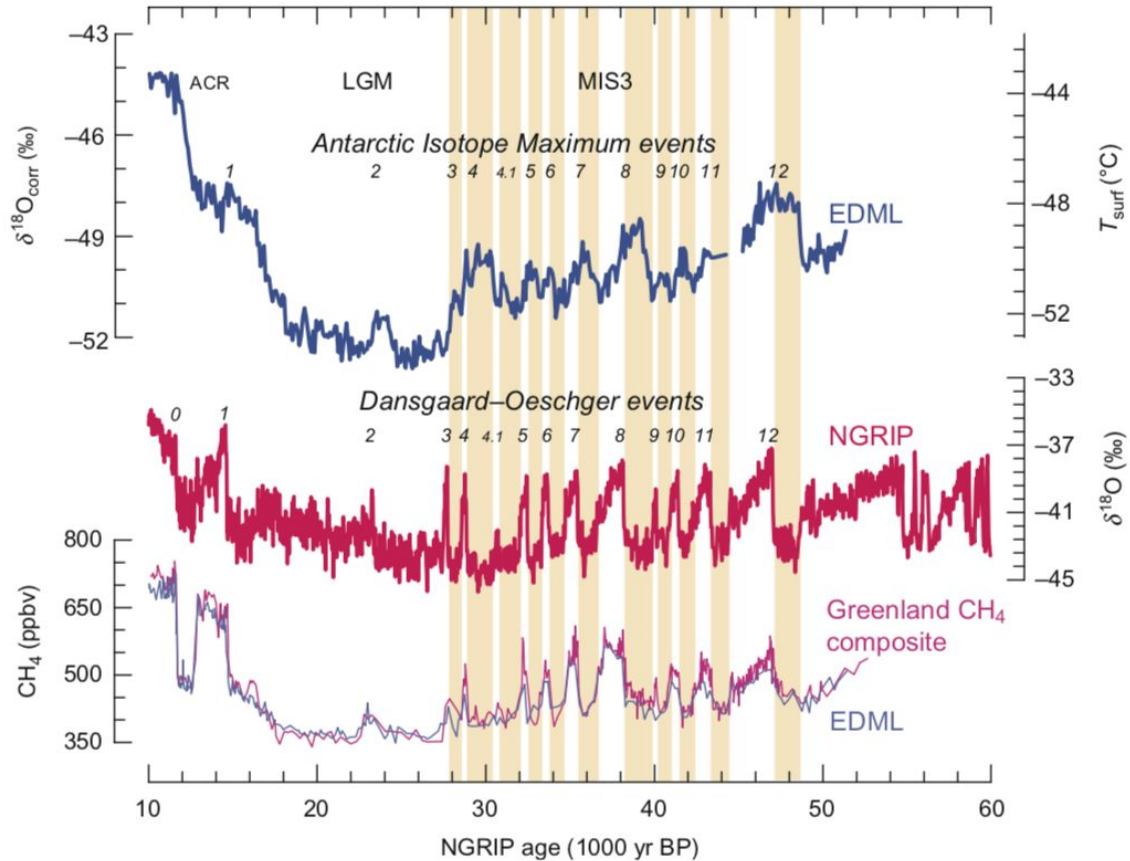


Pre-industrial CO₂;
LGM orbital parameters

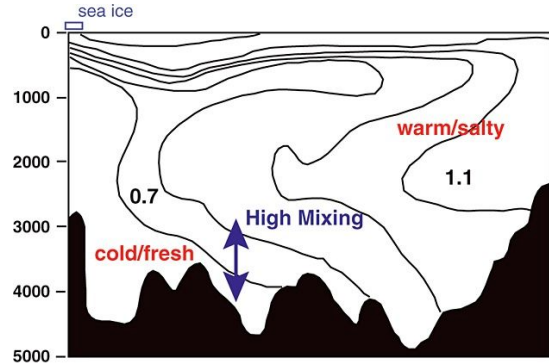


LGM CO₂ and
orbital parameters

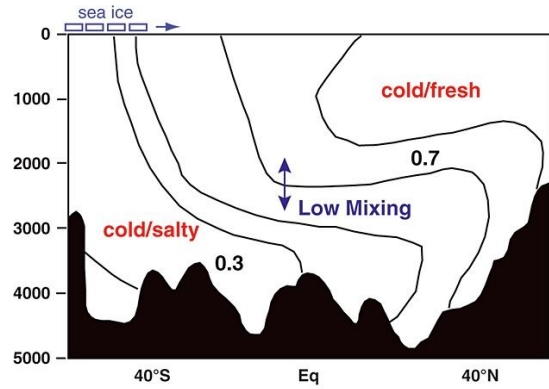
Abrupt climate change during glacial periods: Dansgaard-Oeschger events



Atlantic Meridional Overturning Circulation

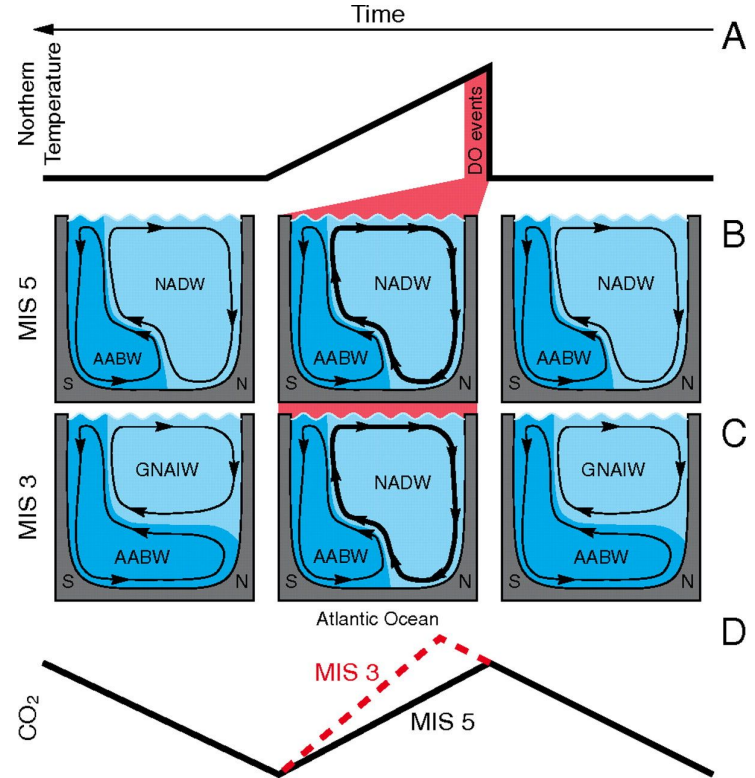


A. Modern Atlantic



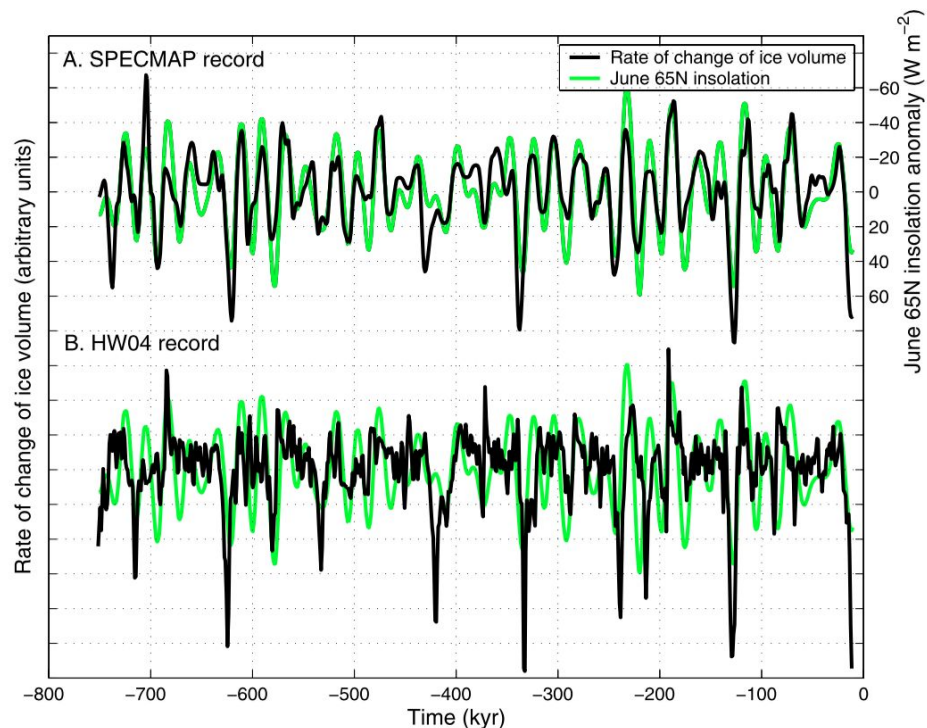
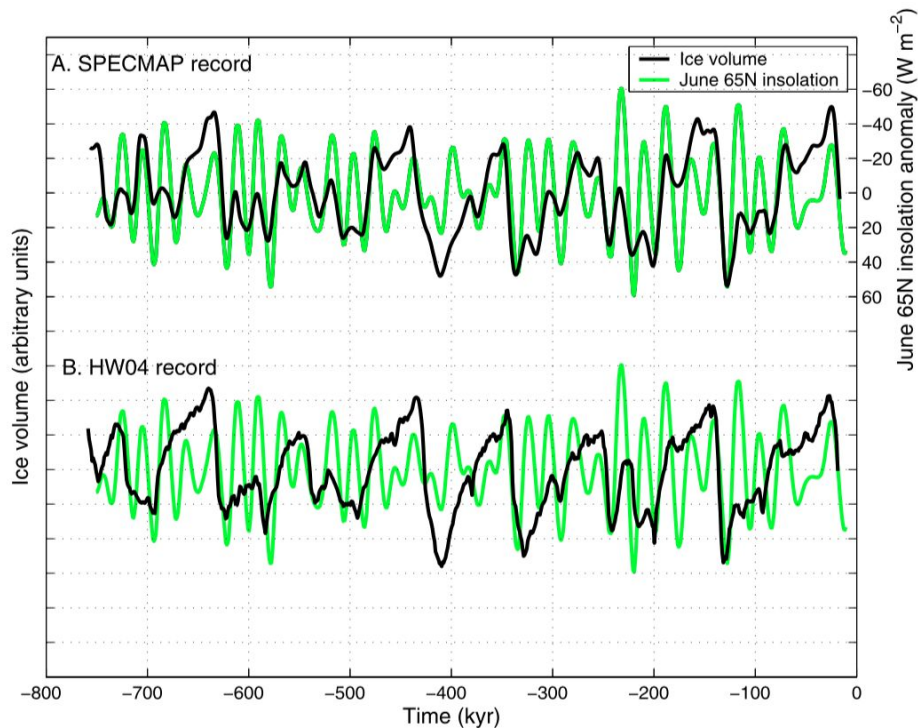
B. Glacial Atlantic

Adkins (2013)

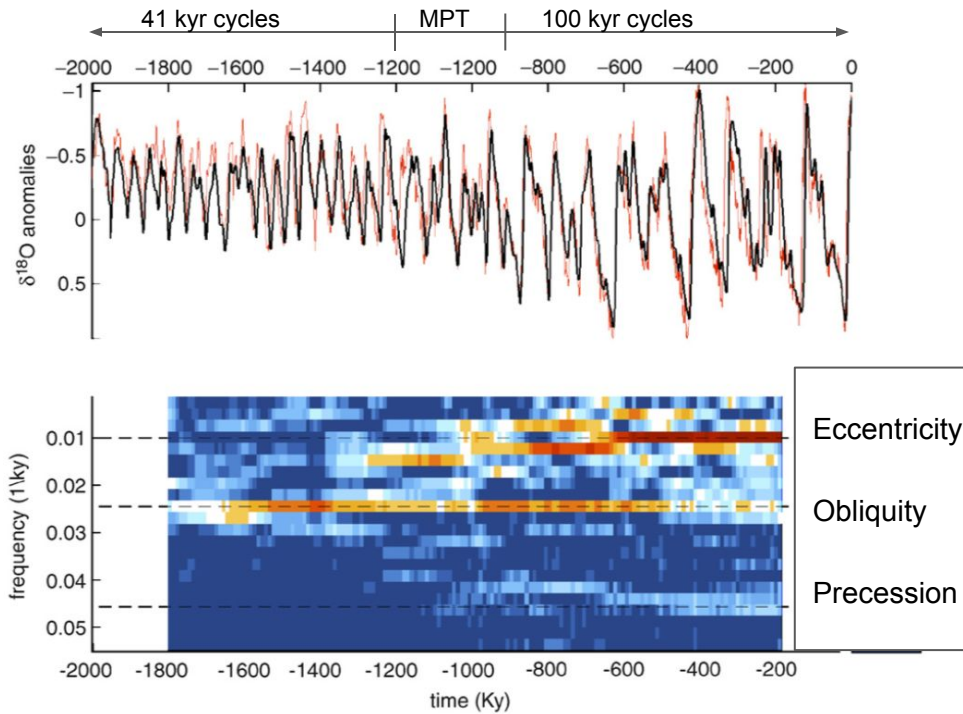


Bereiter et al. (2012)

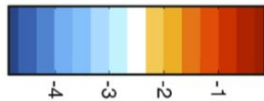
So what drives ice ages?



Roe (2006). In Defense of Milankovitch

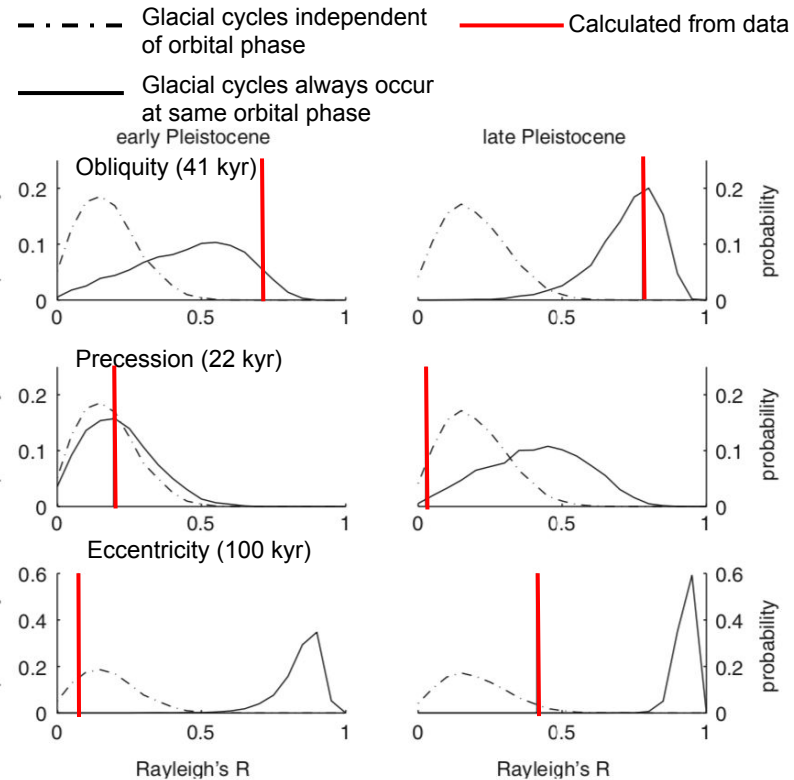


low spectral power
(small signal)



high spectral power
(large signal)

Huybers (2007). Glacial Variability over the Last Two Million Years



Takeaway: Despite appearance of a change from obliquity (41 kyr) to eccentricity (100 kyr) pacing, *glacial cycles are controlled by obliquity*. The appearance of 100 kyr cycles is actually the result a series of 80 kyr and 120 kyr glacial cycles.

Summary

Greenhouse effect is required to explain global temperatures, but the long-term cooling and decline in CO₂ since the Eocene are not well understood.

Feedbacks between ice sheets, oceans, and climate are required to explain records of climate and ice volume.

Because of these feedbacks, the poles experience more dramatic climate fluctuations than the global mean.

The Atlantic Meridional Overturning Circulation links climates of the northern and southern hemispheres.

Quaternary ice age cycles are controlled by obliquity, despite appearance of 100 kyr cycles since 1 Ma.



Ice sheet and climate models

What's a numerical model?

Just a bunch of code that solves (usually) differential equations.

Computers are bad at doing calculus, but they are very good at doing arithmetic.

We discretize these equations and pretend calculus was never invented.

For example:
$$\frac{\partial g}{\partial x} = \frac{g(x + \Delta x, y) - g(x - \Delta x, y)}{2\Delta x}$$

This is different from a statistical model, which derives empirical relationships from data to make predictions instead of using physics*.

*gross generalization

How do we use numerical models?

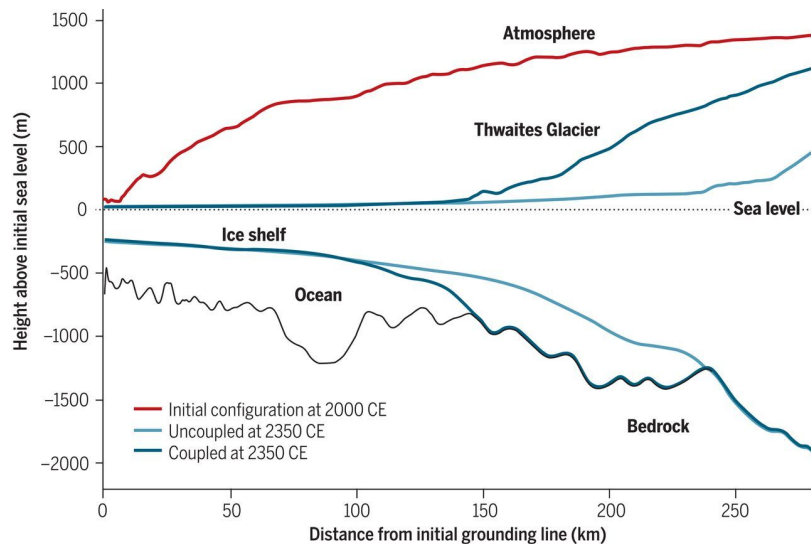
“All models are wrong. Some models are useful.” –

George Box

Using models, we can:

- Examine the sensitivity of a system to elements that are hard to measure or predict in the real world.
- Examine the strength of feedbacks.
- Fill in the gaps between data points (both in space and time).
- Explore scenarios. For example: “What might happen to the WAIS if temperatures exceed the 2° C mark?”

But models are limited by our knowledge of physics and boundary conditions, as well as computational expense (think of calving).



Larour et al. (2019): Slowdown in Antarctic mass loss from solid Earth and sea-level feedbacks

A hierarchy of models, based on approximation to the stress balance

Full Stokes:

Uses all stresses in the stress balance eqs

$$\begin{aligned}\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + \rho g_x &= 0 \\ \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + \rho g_y &= 0 \\ \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z &= 0\end{aligned}$$

Higher-order (or Blatter-Pattyn):

Neglects shear in the z-component of stress balance.

$$\begin{aligned}\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + \rho g_x &= 0 \\ \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + \rho g_y &= 0 \\ \cancel{\frac{\partial \sigma_{zx}}{\partial x}} + \cancel{\frac{\partial \sigma_{zy}}{\partial y}} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z &= 0\end{aligned}$$

A hierarchy of models, based on approximation to the stress balance

Hybrid:

Uses a combination of two stress balances. (note: these aren't literally being added together, but combined in a more sophisticated way).

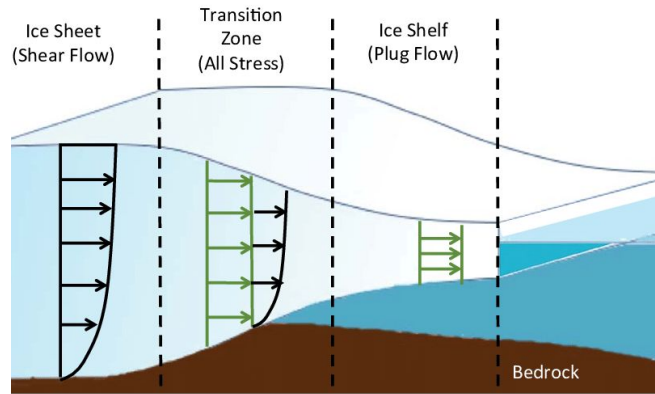
PSU model (Pollard & DeConto) is a hybrid model. This is considered the optimal trade-off between accuracy computational cost for long-term, continent-scale simulations.

$$\begin{array}{l} \cancel{\frac{\partial \sigma_{xx}}{\partial x}} + \cancel{\frac{\partial \sigma_{xy}}{\partial y}} + \frac{\partial \sigma_{xz}}{\partial z} + \rho g_x = 0 \\ \cancel{\frac{\partial \sigma_{yx}}{\partial x}} + \cancel{\frac{\partial \sigma_{yy}}{\partial y}} + \frac{\partial \sigma_{yz}}{\partial z} + \rho g_y = 0 \\ \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z = 0 \end{array}$$

Shallow Ice Approximation: Neglects stretching and horizontal shear. Generally considered too simple to be useful on its own, except for a few specific applications. Does not represent fast flow well.

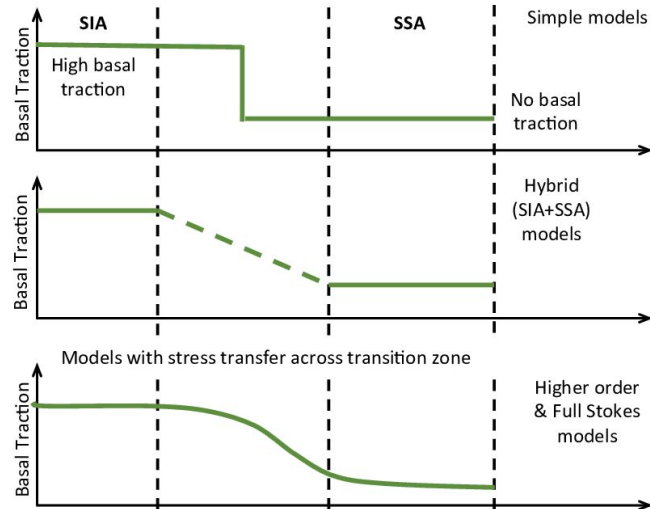
$$\begin{array}{l} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \cancel{\frac{\partial \sigma_{xz}}{\partial z}} + \rho g_x = 0 \\ \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \cancel{\frac{\partial \sigma_{yz}}{\partial z}} + \rho g_y = 0 \\ \cancel{\frac{\partial \sigma_{zx}}{\partial x}} + \cancel{\frac{\partial \sigma_{zy}}{\partial y}} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z = 0 \end{array}$$

Shallow Shelf or Shelfy-Stream Approximation: Neglects vertical shear. Assumes plug flow. Still widely used because it works pretty well for fast-flowing regions. (e.g., Joughin et al., 2014. Marine Ice Sheet Collapse Potentially Under Way for the Thwaites Glacier Basin, West Antarctica)



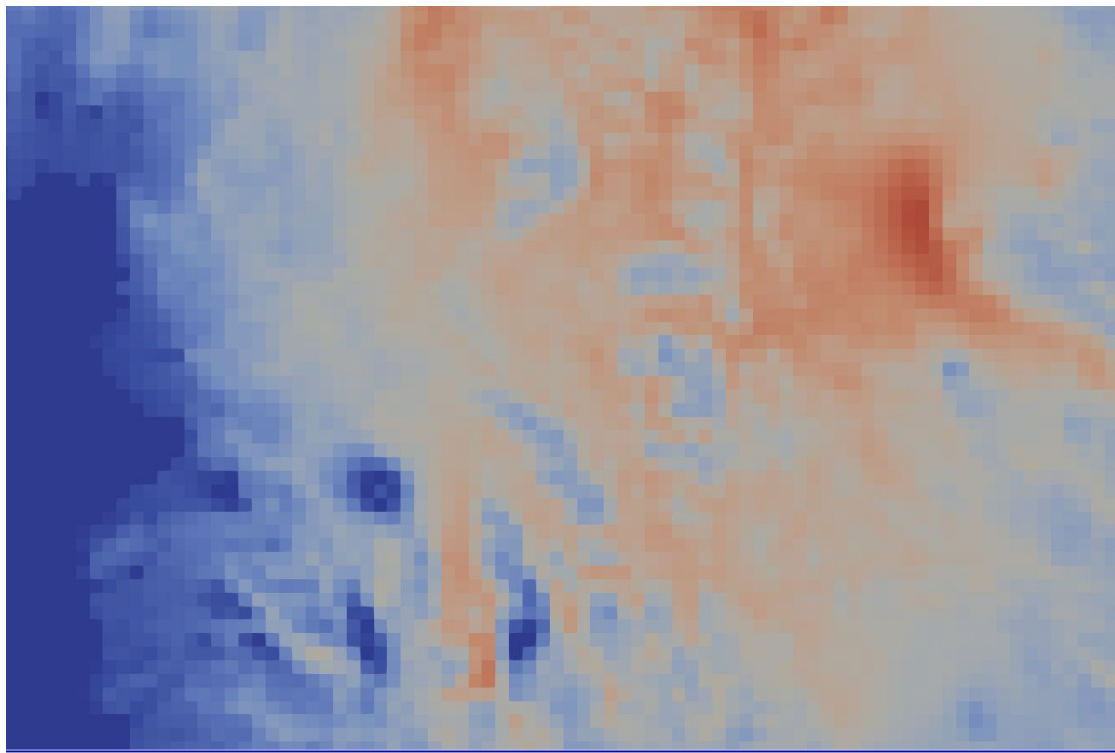
Models with no longitudinal stress transfer across transition zone

Increasing computational
expense and accuracy



Nowicki, S., and H. Seroussi. 2018.
Projections of future sea level
contributions from the Greenland and
Antarctic Ice Sheets: Challenges
beyond dynamical ice sheet modeling.

Structured grids



PSU ice sheet model. Ice thickness of Thwaites Glacier shown.

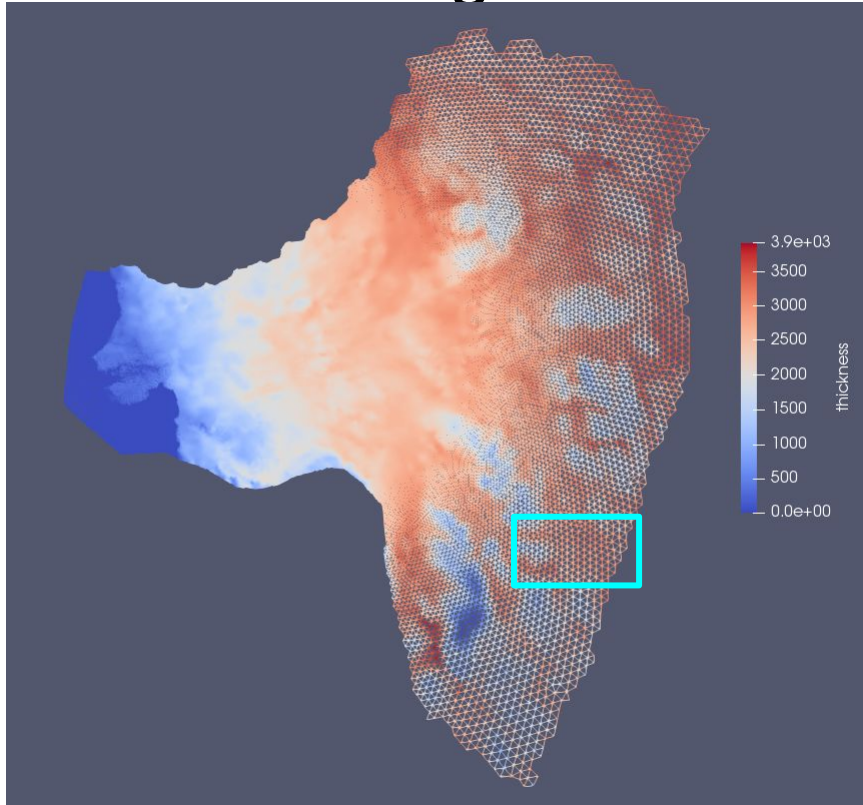
Pros:

- Very simple to create and understand
- Small memory footprint
- Effect of resolution on results is easy to quantify and understand

Cons:

- Limited to rectangular domains
- Not great at handling complex geometries
- Difficult to transition smoothly from low to high resolution

Unstructured grids

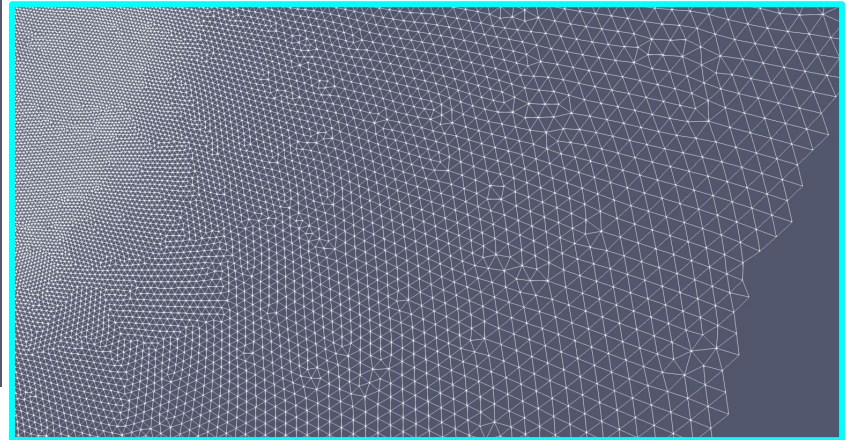


Pros:

- Allow for complex, non-rectangular geometries
- High resolution where needed and low resolution where you can get away with it
 - Smoothly transition between high and low res

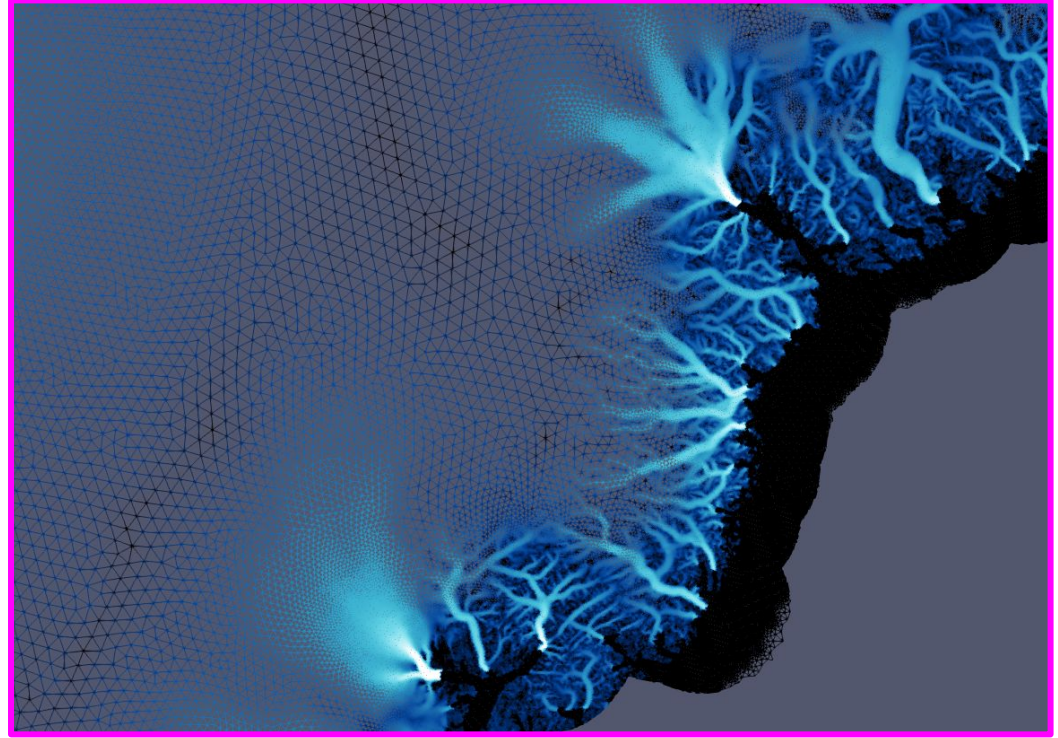
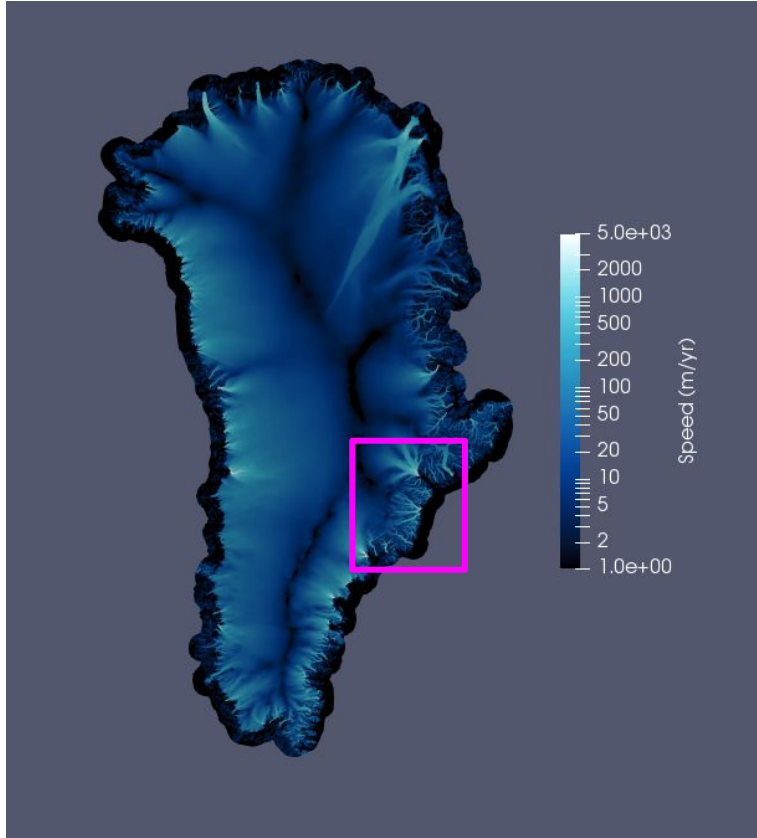
Cons:

- Often a huge undertaking to create
- Often makes output harder to analyze
- Sometimes unpredictable effects of mesh geometry on solution
- Takes many fields to define → requires lots of memory



MALI 1–8km mesh for Thwaites

Unstructured grids



MALI 1–10km mesh for Greenland

Adaptive mesh refinement

Some ice sheet models have the ability to change the mesh through time to track fast flow and the grounding line.



Modeling decisions and tradeoffs

Experimental design

- Length of model runs
- Domain size and resolution of model runs
- Number of dimensions
- Number of model runs

Want to run 100s of simulations for all of Antarctica for a million years? Use a hybrid model.
Want to run a few simulations of Thwaites Glacier for fifty years? Use higher-order or full Stokes.

- Model capabilities. Do you need:
 - adaptive mesh refinement?
 - unstructured mesh?
 - optimization?
 - basal erosion?
 - subglacial hydrology?
 - cliff failure and hydrofracture?
 - parallelization (able to run one simulation across multiple processors)

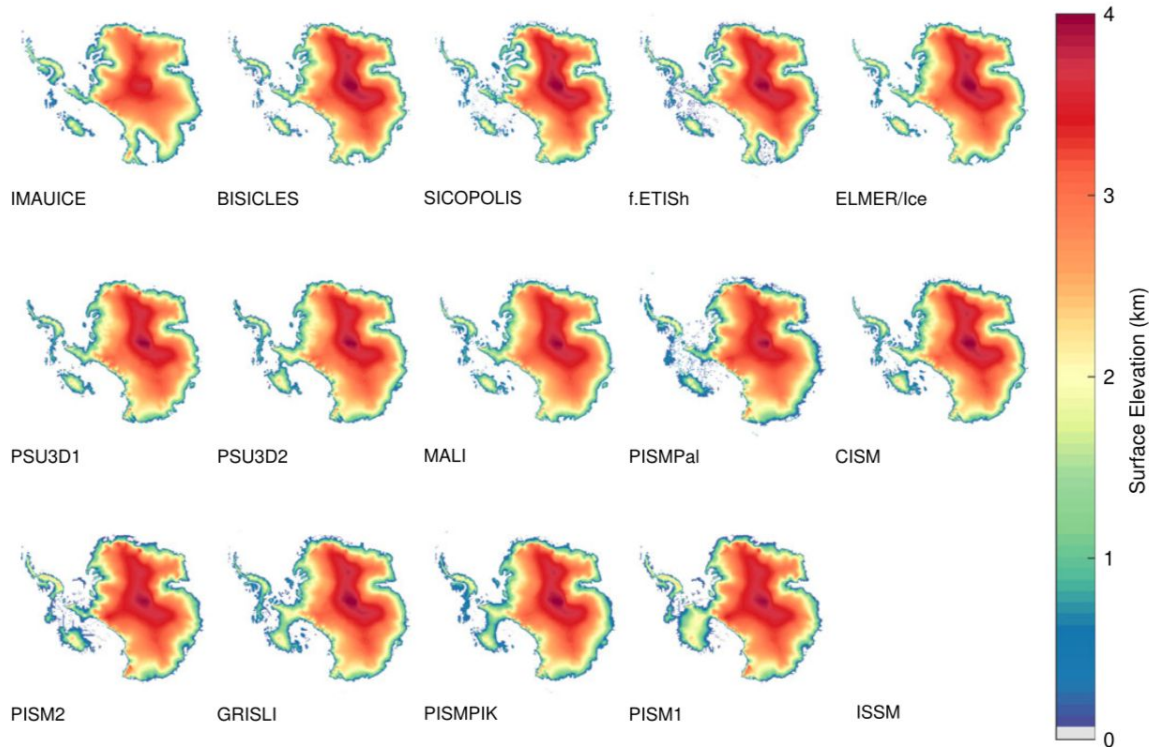
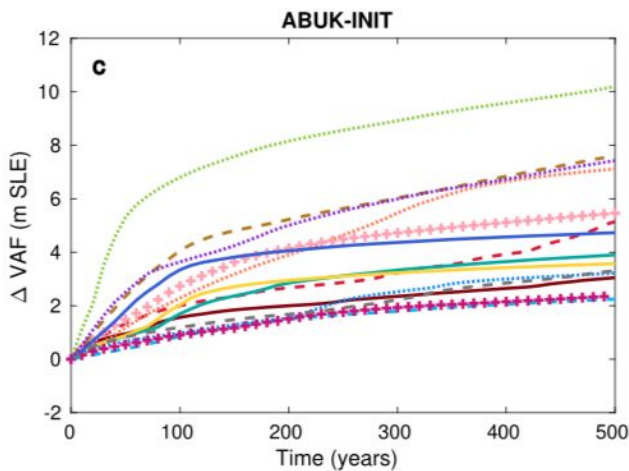
Resources

- Your computing resources. Do you have a supercomputer (higher-order or full stokes), or are you running this on your laptop (SIA, SSA, Hybrid)?
- Your computing, coding, and data analysis skills.
- Community. Is there an active community, or are you going to be the only person using this model in a few years?
- Your time.
- Your sanity.

Ice sheet models on the importance of ice shelf buttressing

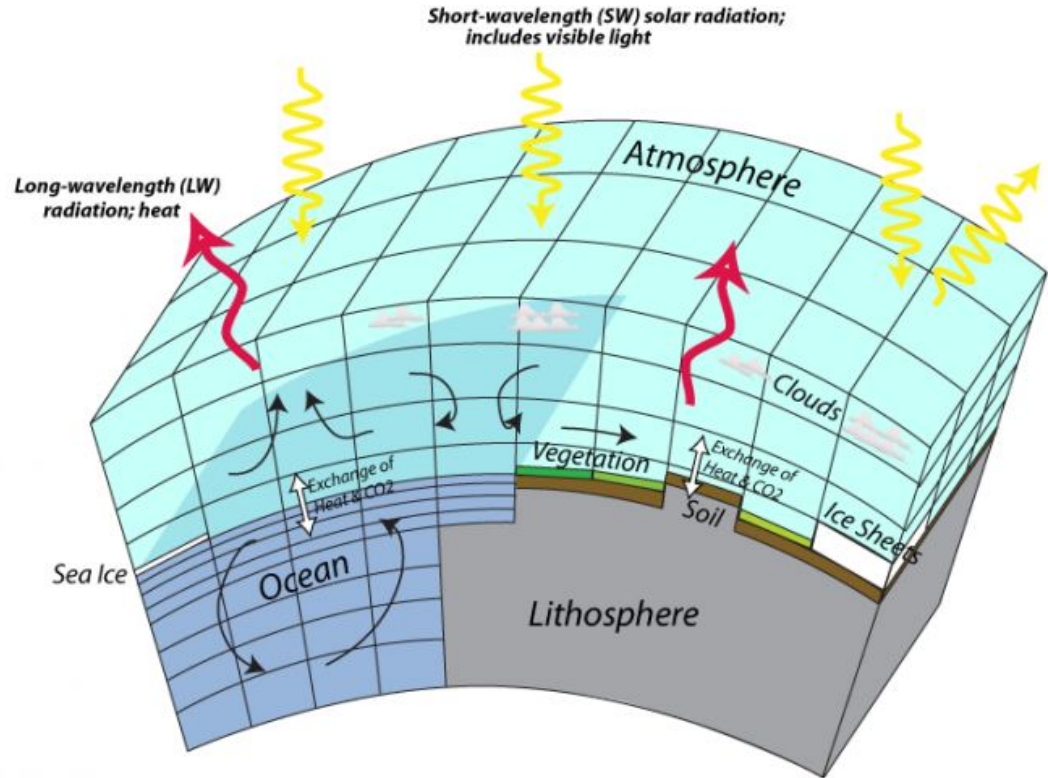
ABUMIP: The Antarctic
BUttrressing Model
Intercomparison project.

Remove ice shelves, run
out 500 years with
modern climate.

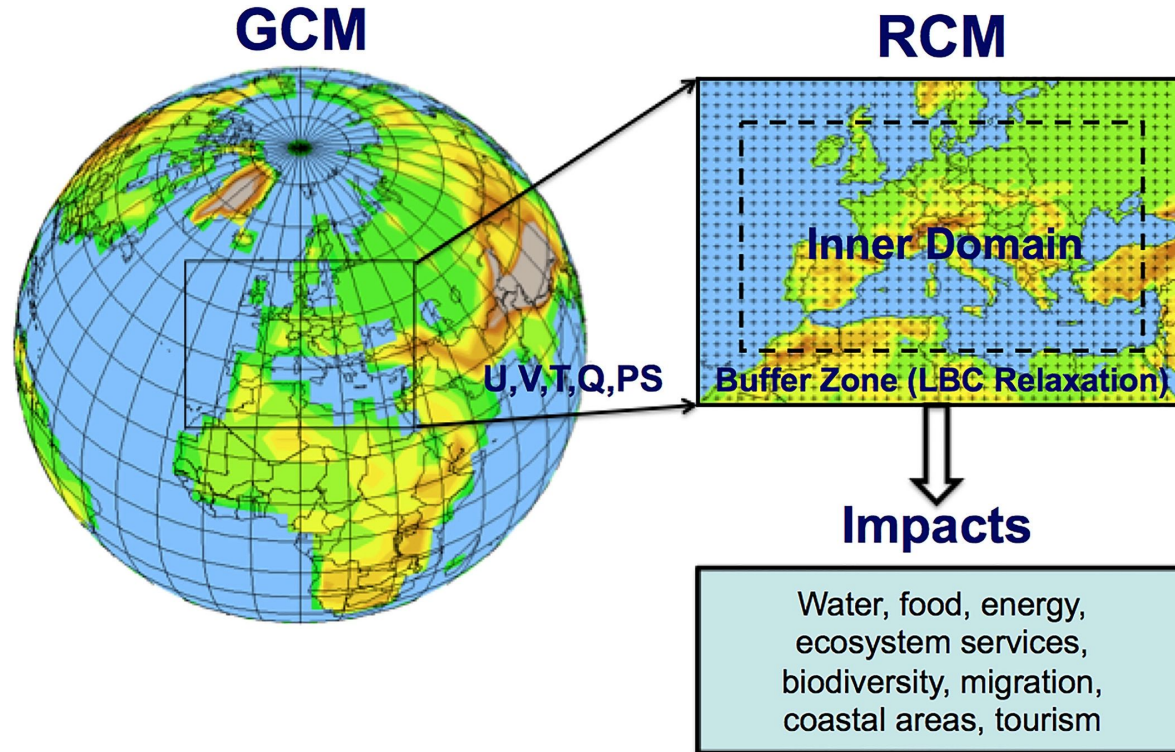


General circulation models describe the flow of the oceans and/or atmosphere, as well as transfer of energy and gases between land, ice, ocean, and atmosphere.

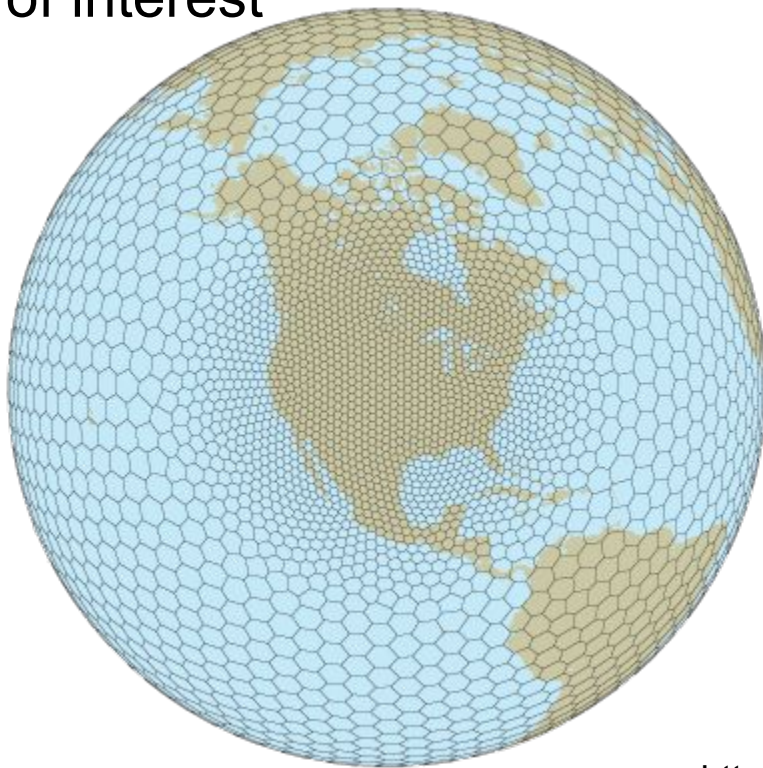
At present, atmospheric GCMs can achieve ~30 km resolution, with 30–50 vertical layers.



Dynamic downscaling from global to regional



Variable resolution meshes allow for global simulations with high resolution in areas of interest



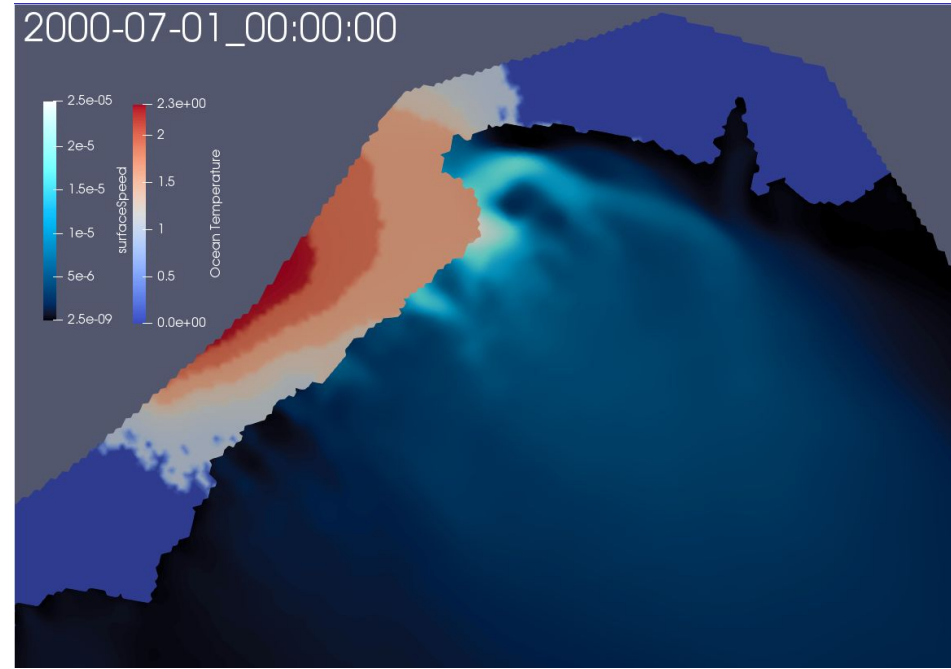
<https://mpas-dev.github.io/atmosphere/atmosphere.html>

Use of climate models in ice sheet modeling

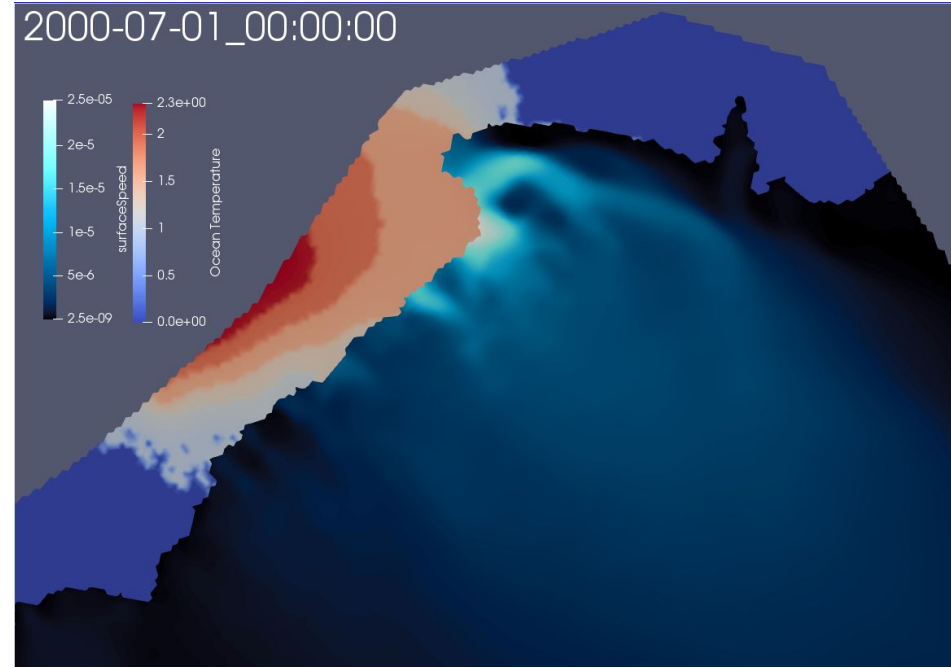
Coupled atmosphere-ocean GCMs only recently run over long paleoclimate timescales (c.f. Tigchelaar et al., 2018)

Ice sheet models are generally forced by regional climate and/or ocean model output.

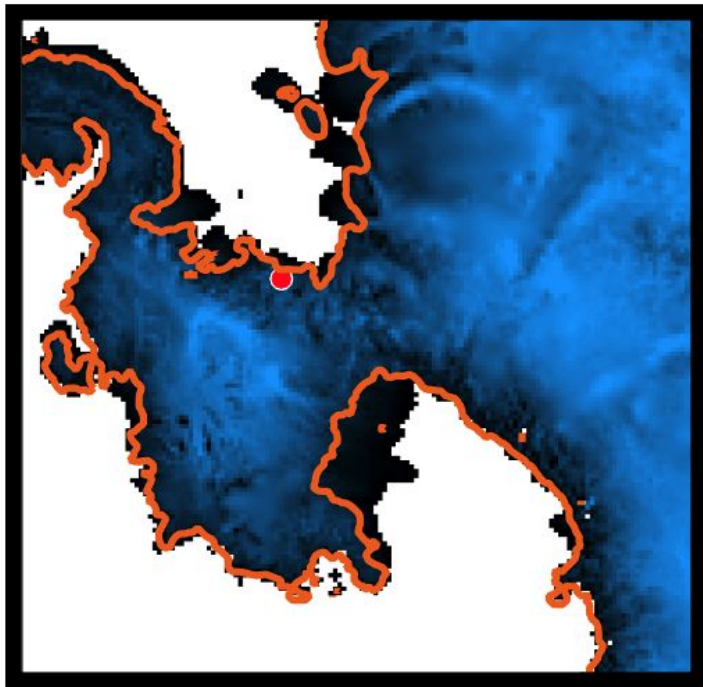
True coupling between ocean sheets, ocean, and atmosphere in models is extremely difficult.



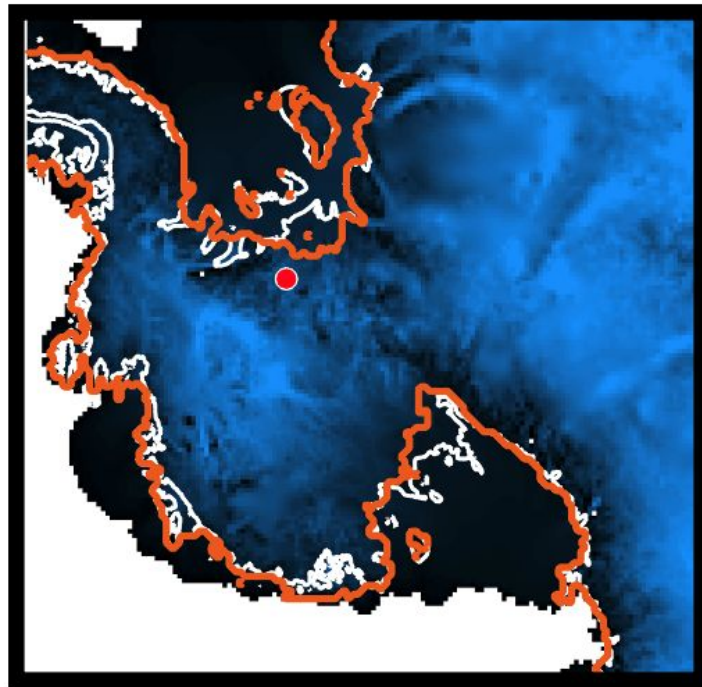
Use of climate models in ice sheet modeling



Effects of climate forcing on simulations of the
Last Interglacial in West Antarctica



My run with parameterized climate
(Hillebrand, 2019)



Forced by a coupled
atmosphere-ocean GCM
(Tigchelaar et al., 2018)